Volume IV. *May–June, 1897.* *Number 6.*

THE

PHYSICAL REVIEW.

ON THE ABSORPTION OF THE EXTRAORDINARY RAY IN UNIAXAL CRYSTALS.

By Oscar M. Stewart.

DICHROISM was observed in the early part of this century by Arago[1] in certain specimens of barite, and by Biot[2] in tourmalin. This variation of color was explained by Biot as being due to the unequal absorption of the ordinary and extraordinary rays. Brewster[3] soon after published a list of some sixty uniaxal and biaxal crystals which presented the same characteristic, although in no case was the dichroism so marked as in tourmalin. More recent researches in the ultra-violet[4] and infra-red[5] have shown that other crystals, Iceland spar for example, apparently entirely transparent to the eye, show this selective absorption.

This selective absorption appeared for a long time anomalous, the extraordinary ray alone showing marked variations for different directions of vibration. On account of its bearing on the laws of double refraction the phenomenon has been given considerable attention.

More recently it has been established beyond all doubt by the experiments of Schwebel[6] and Becquerel,[7] and confirmed by all

[1] Journal de Physique, t. 90, 41, 1820. Also Becquerel, Annales de Chimie et de Physique, 14, 170, 1888, and Mascart's Traité d'Optique, t. 2, 205.

[2] Bulletin de la Société Philomathique, pour 1819, 109 et 132.

[3] Edinburgh Philosophical Journal, p. 348, 1817. Also Phil. Trans., p. 11, 1819.

[4] Agafonoff, Archives des Sciences Phys. et Nat. t. 2, p. 349, 1896.

[5] Merritt, Physical Review, Vol. II., p. 424, 1895.

[6] Zeitschrift für Krystallographie, 7, 153, 1883.

[7] Annales de Chimie et de Physique (6), 14, 170, 1888.

the later investigations, that the absorption for any given wave length depends upon the direction of vibration. Thus for the ordinary ray, which always vibrates in a direction at right angles to the optic axis, the absorption is the same for all directions of propagation. But the angle made with the optic axis by the direction of vibration of the extraordinary ray varies from 0° to 90° for the different directions of propagation of the ray. Since, when this angle is 90°, the extraordinary vibration coincides with the ordinary, we should expect the absorption of the extraordinary ray to vary from a minimum or maximum, as the case may be, to an identity with the absorption of the ordinary ray, as the direction of propagation of the ray is varied. This is in accord with other properties of uniaxal crystals, as, for example, the variation in the velocity of light in the crystal.

The study of this variation in the absorption of the extraordinary ray with the direction of vibration has given rise to a number of theoretical and experimental investigations.

CURRENT THEORIES.

According to the law of thickness, if A_0 be the initial amplitude and A be the resultant amplitude of vibration, we have for any absorbing medium,

$$A = A_0 a^x, \tag{1}$$

where x is the thickness of the medium traversed and a is the factor called the *coefficient of transmission.*

It has been shown that the law of thickness may also be expressed by

$$A = A_0 \epsilon^{-mx}, \tag{2}$$

where m is the factor called the *coefficient of absorption.*

If expressed as a ratio between intensities rather than amplitudes, these formulas become

$$I = I_0 a^{2x}, \quad I = I_0 \epsilon^{-2mx}. \tag{3}$$

Obviously, $\log_\epsilon a = -m.$

The factors a and m, for any given wave length, are constants for all isotropic media, and for the ordinary ray in uniaxal crystals.

In 1888, Becquerel[1] announced that the law governing the absorption of the extraordinary ray could, for a given wave length, be expressed by the equation

$$A = A_0(a_o \sin^2 \alpha + a_e \cos^2 \alpha),$$

or

$$a = a_o \sin^2 \alpha + a_e \cos^2 \alpha, \qquad (4)$$

where α is the angle made by the direction of vibration with the optic axis, a_o the coefficient of transmission of the ordinary ray, a_e the coefficient of the extraordinary ray when its direction of vibration is parallel to the optic axis, and a the coefficient for the direction of vibration making an angle α with the optic axis.[2]

For biaxal crystals, the law was

$$A = A_0(a\lambda^2 + b\mu^2 + c\nu^2),$$

where a, b, and c are the principal coefficients of transmission, and λ, μ, and ν the direction cosines of the vibration referred to the three principal optical directions.

Becquerel believed that he had experimentally verified this law by observations in the visible spectrum taken with a spectrophotometer.

Carvallo[3] has pointed out that Becquerel's law does not conform to the law of thickness. As a result of some experimental work Carvallo came to the conclusion that it was the law of thickness that was at fault, and announced that he had discovered a new phenomenon, "*de signification claire et d'importance capitale.*" A few weeks later Potier[4] showed that it was Becquerel's law, and not the law of thickness, that was at fault. Carvallo, afterwards, came to the same conclusion.

Mallard[5] has developed what has since been called the *ellipsoid of absorption*, or sometimes the *inverse ellipsoid of absorption*. His law may be expressed by the formula

$$m = m_o \sin^2 \alpha + m_e \cos^2 \alpha, \qquad (5)$$

[1] *loc. cit.*

[2] The direction of vibration is the direction as taken in the Fresnel theory.

[3] Comptes Rendus, t. 114, 661, 1892; also, Ann. de Chim. et de Phys. (7), t. 7, 58.

[4] Comptes Rendus, t. 114, 874.

[5] Traité de Cristallographie, 1884, t. 2, 350; Mascart's Traité d'Optique, t. 2, 208.

for uniaxal crystals, and for biaxal crystals by

$$m = m_1\lambda^2 + m_2\mu^2 + m_3\nu^2, \tag{6}$$

the notation being the same as before.

In equation (6) the square root of m is the perpendicular, parallel to the direction of vibration, let fall from the center on the tangent plane to the ellipsoid

$$\frac{x^2}{m_1} + \frac{y^2}{m_2} + \frac{z^2}{m_3} = 1.$$

Mascart[1] has shown that when the absorption is small the formulas of Becquerel and Mallard are identical. The equation

$$\log_\epsilon a = -m$$

may be written

$$a = \epsilon^{-m}.$$

Expanding this equation and neglecting m^2 and higher powers, we have

$$a = 1 - m.$$

When this value of a has been substituted in equation (4), an equation identical with equation (5) will be obtained. Thus we see that when m is so small that its square may be neglected the formulas of Becquerel and Mallard are identical.

In 1895, Moreau[2] developed a theory which gives an unsymmetrical relation between the principal coefficients of absorption, and the asymmetry increases as the difference between the principal indices of refraction increases. But with crystals for which the indices are not very different his equation becomes

$$\frac{m}{n} = \frac{m_o}{n_o}\sin^2\alpha + \frac{m_e}{n_e}\cos^2\alpha, \tag{7}$$

where n is the index of refraction for the angle α, and n_o and n_e the maximum and minimum indices of the extraordinary ray. In this form it was verified by data taken by Camichel with tourmalin, in which the indices were practically the same.

Camichel,[3] working with an ingenious spectro-photometer, was

[1] Traité d'Optique, t. 2, 208.

[2] Comptes Rendus, t. 119, 327, et t. 120, 258 et 602.

[3] Annales de Chim. et de Phys., t. 4, 433, 1895. Séances de la Soc. Franç. de Phys., 50, 1895. Also Journal de Physique, t. 4, 149.

able to make more exact determinations than had been previously made. For uniaxal crystals, he worked with different thicknesses and varieties of tourmalin. His results for yellow light confirm Mallard's theory and contradict that of Becquerel.

Carvallo[1] has recently developed a theory based on the equations of Boussinesq,[2] which may be derived[3] from the well-known equations of Helmholtz. He obtains as the equations of the extraordinary ray, for a given wave length,

$$\frac{1}{n^2} = \frac{1}{n_o^2}\sin^2\alpha + \frac{1}{n_e^2}\cos^2\alpha, \tag{8}$$

and

$$\frac{m}{n^3} = \frac{m_o}{n_o^3}\sin^2\alpha + \frac{m_e}{n_e^3}\cos^2\alpha, \tag{9}$$

where the notation is the same as that already used. The first is the equation of the variation of the refractive index of the extraordinary ray, the index being defined as the ratio of sines or of wave velocities, not as the ratio of ray velocities. His second equation reduces to the form of Mallard, equation (5), for substances whose principal indices are practically equal.

Carvallo's experiments were conducted with tourmalin, whose principal indices differed by about one per cent. On account of this small variation of the refractive indices he was unable to experimentally distinguish between his theory and that of Mallard. His data, obtained with a linear thermopile at $\lambda = 1.98\,\mu$, confirm the theory of Mallard and contradict that of Becquerel.

In 1884, Voigt[4] developed a theory, the equations of which reduce to the same form as those of Carvallo. Voigt uses as an absorption coefficient one first used by Cauchy. It is defined by expressing the law of thickness by the equation

$$A = A_0 \epsilon^{\frac{\kappa x}{\tau q}},$$

where x is the thickness, κ a factor called the *extinction coefficient*, $\frac{1}{2\pi\tau}$ the frequency of vibration, and q the velocity of wave propa-

[1] Ann. de Chim. et de Phys. (7), t. 7, 58, 1896.

[2] Poincaré Theorie Math. de la Lumière, 208.

[3] Comptes Rendus, t. 112, 521, 1891.

[4] Wiedemann's Annalen, 23, 577. Also, Kompendium der theoretischen Physik, 2, 708.

gation in the medium. The product of this extinction coefficient and the refractive index is called the *absorption coefficient.*

Obviously
$$m = \frac{\kappa}{\tau q} = \frac{2\pi\kappa}{l} = \frac{2\pi s}{ln}, \tag{10}$$

where s is the Cauchy absorption coefficient, and l the wave length in the medium. Voigt gives the following equations for the extraordinary ray, —

$$q^2 = o^2 \sin^2\alpha + e^2 \cos^2\alpha, \tag{11}$$

and
$$2\tau\kappa = \frac{C_o \sin^2\alpha + C_e \cos^2\alpha}{o^2 \sin^2\alpha + e^2 \cos^2\alpha}, \tag{12}$$

where q and κ are the velocity of propagation in the medium and the extinction coefficient, respectively, of the vibration making an angle α with the optic axis; o, e, C_o, and C_e are constants for the wave length whose frequency is $\frac{1}{2\pi\tau}$. If equation (11) be divided by the square of the velocity of propagation in air, it reduces to equation (8), the first of the equations of Carvallo.

Equation (12) may be written

$$2\tau\kappa = \frac{1}{q^2}(C_o \sin^2\alpha + C_e \cos^2\alpha),$$

and as
$$n = \frac{q_a}{q},$$

where q_a is the velocity in air, we have, from the relations given by equations (10),

$$\frac{\kappa q^2}{\tau q_a^3} = \frac{1}{2\tau^2 q_a^3}(C_o \sin^2\alpha + C_e \cos^2\alpha),$$

or
$$\frac{m}{n^3} = \frac{1}{2\tau^2 q_a^3}(C_o \sin^2\alpha + C_e \cos^2\alpha).$$

Solving this equation for the constants on the right-hand side by giving α values of 0° and 90° respectively, we have

$$\frac{m}{n^3} = \frac{m_o}{n_o^3}\sin^2\alpha + \frac{m_e}{n_e^3}\cos^2\alpha,$$

or equation (9), the second of Carvallo's equations. Thus we see that Carvallo has been anticipated in his theory by a dozen years.

It is thus seen that there is a considerable difference in the theories, and that in some cases the experimental evidence has

been conflicting. Becquerel's formula may be regarded as discredited, both by theory and by all recent experiments. The other theories, namely those of Mallard, Voigt or Carvallo, and Moreau, as far as experimental evidence is concerned, have equal claims. Their experimental verification has been carried out with data obtained from crystals whose principal refractive indices differed very little. As we have seen, these formulas are practically identical for such crystals.

The present work was undertaken with the thought of attempting to experimentally distinguish between these different theories. The problem was brought to the attention of the writer by Pro-

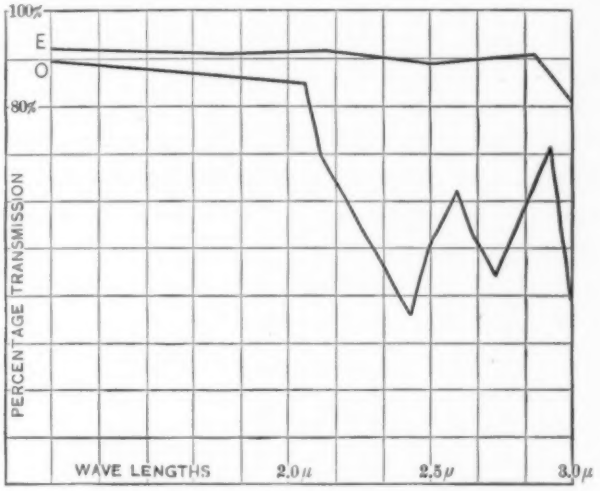


Fig. 1.

fessor Merritt, and the writer is further indebted to him for his timely suggestions at various stages in the work.

As a substance more nearly filling the required conditions than those previously used, Iceland spar was chosen for the work. The difference in the maximum and minimum refractive indices is relatively great, and in the infra-red it presents a suitable absorptive power. As may be seen from the results of Professor Merritt,[1] partially reproduced in Fig. 1, for wave lengths between 2.0 μ and 2.8 μ there is a marked difference in the absorption of the

[1] PHYSICAL REVIEW, Vol. II., p. 424. Also Wiedemann's Annalen, 55, 49, 1895.

extraordinary ray for different directions of vibration.[1] Yet the absorption is not great enough to modify the reflection or the refractive indices.

For this region Iceland spar is seen to be an almost ideal substance for the present investigation. The great difference in the principal indices should make the formulas of Mallard, Voigt or Carvallo, and Moreau quite different. And the difference between the absorption of the ordinary ray, and that of the extraordinary ray corresponding to the maximum transmission, presents a variation great enough to make a considerable change in the transmission of the extraordinary ray, when the direction of the vibration is varied.

APPARATUS AND ADJUSTMENTS.

A number of plates of Iceland spar were obtained, whose faces made different angles with the optic axis. These plates were of the purest variety, showing no absorption for the visible portion of spectrum. By means of the spectro-bolometer, measurements were made in the usual manner.

A diagram of the apparatus, as finally used, is shown in Fig. 2.

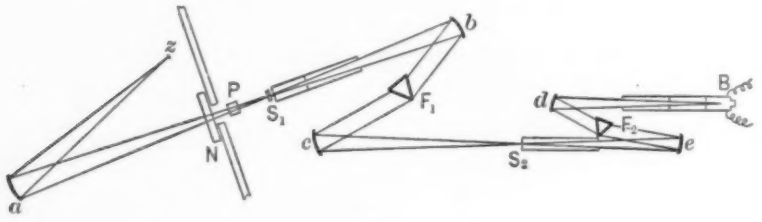


Fig. 2.

Two spectrometers were connected "in series" to obtain the greatest possible purity of the spectrum. It has been shown[2]

[1] The curves shown in Fig. 1 are from data taken by Professor Merritt with a plate 2.98 mm. thick, the face of which was cut parallel to the optic axis. The upper curve in the figure was obtained when the vibrations were parallel, and the lower when they were perpendicular, to the optic axis. They correspond to the maximum and minimum transmission of the extraordinary ray, the minimum being in this case identical with the ordinary ray. Values, as given by the curves, are not corrected for reflection.

[2] Merritt, *loc. cit.*

that unless precautions of this kind be taken, the stray light from the more intense portions of the spectrum will vitiate the results obtained. Energy from the intense zirconium lamp z was focused on the slit S_1 by the concave mirror a. The rays were rendered parallel by the concave mirror b, were dispersed by the fluorite prism F_1, and brought to a focus at S_2. To the mirror c was attached a tangent movement, so that the spectrum could be slowly shifted in a direction at right angles to the slit S_2, and any desired portion of the spectrum permitted to fall upon the narrow slit. The rays passing through S_2 were again dispersed by another fluorite prism F_2, and brought to a focus in the plane of the linear bolometer B.

Both prisms were kept set for the minimum deviation of the D line. The prism F_2 was calibrated, and from values of the refractive indices, as obtained by Paschen,[1] the wave lengths could be computed when the angular deviation from the D line had been measured. These angles were read on the circle of the second spectrometer.

At P was the polarizer, which will be explained later. Immediately in front of the slit S_1 was a vertical track, sliding along which was a metallic plate. On this metallic plate was mounted the crystal plate to be examined. By this arrangement the crystal plate could be removed from the incident beam, and brought back accurately to the same position. Both this plate carrier and the screen at N could be operated by cords from the observer's position at the reading telescope.

Metallic mirrors possess two distinct advantages, viz., that they do not absorb the energy, and that their focal length is the same for all wave lengths — that is, when focused for the visible rays they are focused for all wave lengths. The mirrors used were of glass, silvered on the first surface. During the course of the work it was found necessary to resilver them. This was done by the Brashear method,[2] which gave a coating so tenacious that it could be polished with rouge.

[1] Wiedemann's Annalen, 53, p. 812, 1894.

[2] Wadsworth, Zeitschrift für Instrumentenkunde, 22, 1895. Also Astrophysical Journal, Vol. I., p. 252, 1895.

The galvanometer used was one built several years ago by the department mechanician, Mr. Fowler. A description of it has already appeared.[1] The coils used when connected in series gave a resistance of 15 ohms. The moving parts, consisting of mirror, glass connecting rod, and eighteen magnets, weighed 0.095 g. This magnetic system was magnetized by the Wadsworth method.[2] It pointed east and west, and gave a period of seven seconds in the earth's field. In practice this field was weakened until the needle had a period of about twelve seconds, which was found to be the most suitable. The irregular vibrations of the needle, due to the mechanical disturbances of the machinery in the neighborhood, were very annoying. An attempt to remedy this was made by placing the galvanometer on a marble slab that lay on a thick bed of sawdust. The slab had a mass of about 30 Kg. This acted fairly well until the sawdust became packed, when it was as bad as ever. Finally a platform was suspended by three similar and parallel cords from three points in the ceiling. On this platform was placed a slab having a mass of about 15 Kg., and upon this the galvanometer. With this arrangement the galvanometer was practically isolated from the mechanical disturbances, and as far as these were concerned its behavior was highly satisfactory.

The Bolometer.

In the bolometer first used the two strips were of iron, each about 1.5 cm. long and of 1 ohm resistance. The width of the exposed strip, as measured on the circle of the spectrometer, was 1.7′. Later a bolometer was used in which the strips were of platinum. They were made by rolling out a platinum drawn-in-silver microscope cross-hair wire. The diameter of this wire, as given by the manufacturers, was 0.006 inch, and of the platinum core, 0.00022 inch. After being mounted, the silver was removed with acid. Each strip was 9 mm. long, and had a resistance of about 4 ohms. After smoking, the width of the exposed strip, expressed in circular measure, was 1.2′. The thickness of the

[1] E. F. Nichols, PHYSICAL REVIEW, Vol. I., p. 1; also Nichols, The Galvanometer, pp. 75, 76, Figs. 44 and 45.

[2] Phil. Mag., s. 5, 38, p. 482, 1894.

strip was very much less, approaching the dimensions of light waves. The platinum bolometer, notwithstanding that it was narrower and shorter than the iron one, was much more sensitive. This was probably due to its lower heat capacity and higher resistance. Both of these excellent bolometers were made by the mechanician connected with the laboratory.

The first bolometer used was connected up in the traditional way; that is, the compensating resistances and the resistance of the galvanometer were made equal to the resistance of the strips. This was afterwards changed, for reasons which are given in an article[1] in this number of the REVIEW. The compensating resistances were made about 26 ohms each, and the electromotive force increased until about the same current as before flowed in the strips. The galvanometer was connected to the junctions of the strips with the compensating resistance coils. The gain in sensitiveness of the bolometer from these changes was about 30 per cent. Later the platinum bolometer was connected up in a similar way. For this bolometer the galvanometer coils were connected in multiple series. This did not quite give the most suitable resistance for the galvanometer, but the gain that would result from rewinding the coils was not sufficient to warrant the change.

The current used was furnished by two storage cells, giving a current of about 0.04 ampere through each of the bolometer strips. In constructing the resistances and the usual balancing adjustments, great care was taken to prevent fluctuations in temperature. As an additional precaution, the spectrometers and all the resistances were inclosed in a constant-temperature room. So successful were these attempts that, when in working order, the disturbances of the needle due to the bolometer connections were not greater than the magnetic disturbances, and only a very slow "drift" of the needle was ever present.

The Polarizer.

At first the light was polarized by reflection from glass, and in the case of the visible rays was tested by a Nicol's prism. As

[1] Child and Stewart, p. 502.

the polarizing angle is a function of the wave length, there is the possibility that when one has complete polarization for the visible spectrum, he may not have it for the longer wave lengths. However, the lack of complete polarization for a glass polarizer has been shown[1] to be very small for wave lengths as great as 3.0 μ. But the greatest objection to this method is the great loss of energy. In the visible spectrum the intensity of the beam reflected from the first surface of glass at the polarizing angle, is only 7 or 8 per cent of the incident intensity, and for the infra-red the percentage, neglecting the effect of absorption, is even less.

Figure 1 shows that the extraordinary ray in Iceland spar, out to $\lambda = 2.8$ μ, is transmitted with very little loss, while the ordinary

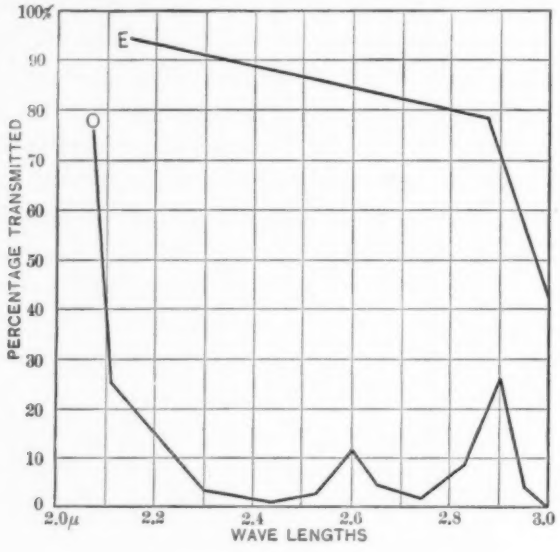


Fig. 3.

ray suffers rather marked absorption. These results were obtained from a plate cut parallel to the axis 3 mm. thick. Now if the thickness of the plate be increased, according to the law of thickness the difference between the absorption of the two rays should rapidly increase. From these results, correcting for loss due to reflection, the transmission for a plate 21 mm. thick, cut parallel to the optic axis, was computed. The approximate values obtained

[1] Merritt, *loc. cit.*

are shown graphically in Fig. 3. These curves do not show the loss due to reflection at the two surfaces; this, however, is relatively small.

Figure 3 shows that such a block of Iceland spar would act for some wave lengths almost as a complete polarizer, possessing the great advantage over glass of giving a polarized beam of about *six* times the intensity.

A cube of Iceland spar, 20 mm. on the edge, with faces parallel and perpendicular to the optic axis, was obtained and set with its optic axis perpendicular to the direction of propagation of the light.[1] The optic axis was placed horizontally, so that the emergent beam was polarized in the plane corresponding to the minimum reflection from the prisms; a precaution in the present arrangement of apparatus probably not important. A correction for the slight error due to incomplete polarization was applied to the readings in the manner explained later.

The Crystal Plates.

In all, eight different plates cut from Iceland spar were used. Each of these crystal plates, which were 1 cm. square on the face, and about 0.4 cm. thick, was mounted on one of the carriers, previously referred to, which moved along in front of the slit S_1. On each of these carriers was a small divided circle. And as the crystal plate was placed over a small opening at the center of this circle, it could be rotated through a known angle about an axis perpendicular to its face. As the crystal and the small opening at the center of the carrier were both larger than the slit S_1, and close to it, no error was introduced by any screening effect on the slit.

With the aid of Professor Gill of the Department of Mineralogy, the angle made by the optic axis with the face of each crystal plate was measured in the following manner. A small fragment having been chipped from one corner of the plate, an almost microscopic portion of one of the natural cleavage planes was revealed. At another corner, a second cleavage plane was similarly obtained.

[1] See *P* in Fig. 2.

With a Fuess No. 2 reflection goniometer the angles between these cleavage planes and the face of the plate were accurately measured. The angle between the optic axis and the face of the plate was computed from these angles and the known constants of Iceland spar.

To set the crystal plates perpendicular to the beam of light an inclined piece of plate glass was temporarily placed in the path of the rays between the mirror a and the slit S_1. By observing in this glass the beam of light reflected from the surface of the crystal plate, the latter was adjusted on its carrier until it was perpendicular.

For the adjustment of the principal planes, two crossed Nicols were temporarily mounted, one in front of the polarizer P and the other between the slit S_1 and the mirror b. By the usual optical method their principal planes were set parallel and perpendicular, respectively, to the principal plane of the polarizer. One at a time the crystal plates were introduced before the slit S_1 and rotated about an axis perpendicular to their faces until their principal planes were parallel or perpendicular to the principal plane of the polarizer, and the readings of the divided circles of the carriers noted.

Table I. gives the thickness of each plate and the angle α made by the optic axis with the face of the crystal.

TABLE I.

Plate.	Thickness in mm.	α	Plate.	Thickness in mm.	α
A	4.03	0° 9′	*E*	3.83	44° 36′
B	3.98	22 4	*F*	3.98	59 41
C	3.76	39 30	*G*	3.64	77 4
D	3.83	44 11	*H*	3.93	82 1

The angle for plate E was not measured. This plate was cut parallel to one of the cleavage planes of the spar, and its angle could therefore not be readily determined by the method used; the angle given in the table is that of the cleavage plane. Plates D and E were obtained from Steg and Reuter, and the remainder from Brashear.

Observations and Results.

The bolometer was set by the circle of the second spectrometer to the angular deviation from the D line corresponding to the desired wave length. The screen N was raised, and by the slow tangent movement the mirror c was turned until the maximum deflection was indicated by the galvanometer. This maximum was well defined, and when attained the slit S_2 admitted the wave length desired. The first throw of the galvanometer when the screen was raised was observed, first when the crystal plate was not in the path of the rays, and next when in place. The ratio of the two throws gave the ratio of the two intensities, or the percentage of transmission.

The plane of polarization was perpendicular to the principal plane of the polarizer. Therefore when the principal plane of the crystal plate was parallel to that of the polarizer, the extraordinary ray was transmitted through the plate. When the plate was set with its principal plane perpendicular to the principal plane of the polarizer, the ordinary ray would be transmitted by the plate. Hence, by a rotation of the plate about an axis perpendicular to its face, either the ordinary or the extraordinary transmission could be obtained. This rotation could easily be effected by the use of the divided circle previously referred to. By the method of adjustment employed the principal plane of the crystal plate was either perpendicular or parallel to that of the polarizer; the method did not permit a distinction to be made. But as the extraordinary ray was known to be always more freely transmitted, the two were readily distinguished by observing the transmission.

In general, ten or more readings were taken alternately, first with the crystal in position before the slit, and when removed. As the galvanometer gave deflections of 150 small scale divisions for the wave lengths used, and as successive readings usually agreed to within one or two per cent, the mean of such a series gave the galvanometer deflections to a rather high degree of accuracy. The error in the reading of the galvanometer throw is probably of much less magnitude than several other errors involved.

The coefficients of transmission are often, for convenience, de-

fined as the ratio of the intensities, and not of amplitudes as given in equations (1) and (2). It is a matter of indifference, since, as one may see from equations (3), the same result is obtained by merely changing the arbitrary unit of thickness by the factor 2. For convenience 3.98 mm. has been used as an arbitrary unit of thickness; and the ratios of the intensities corresponding to this thickness are taken as the coefficients of transmission. If twice the unit given be taken as the unit of thickness, then the coefficients are the ratios of amplitudes.

Owing to the wave lengths used, and the great variation of the refractive index of the extraordinary ray, the usual device for getting rid of the reflection at the face of the crystals by immersing them in some liquid of the same index was not used. The transmission as observed was corrected for reflection by the formula,

$$B_1 = B_2 (1 - r)^2,$$

where B_1 is the percentage of transmission observed, and B_2 the corrected percentage. The factor r is the percentage reflected from the first surface at normal incidence, and may be obtained from the Fresnel formula, as the absorption here is not great enough to modify the reflection. For normal incidence this reduces to the well-known form due to Young,

$$r = \left(\frac{n-1}{n+1}\right)^2, \tag{13}$$

where n is the refractive index. The refractive indices of the ordinary ray and the minimum indices for the extraordinary ray were calculated from the dispersion formula, using the constants for Iceland spar obtained by Carvallo.[1]

In Table II. are given the indices as computed from Carvallo's constants.[2]

[1] Journal de Physique, 9, 265, 1890.

[2] The value of the constant a in the dispersion formula for the ordinary ray, as given in the place cited, is incorrect. As computed by the writer it should be + 0.371069.

TABLE II.

Ordinary.		Minimum extraordinary.	
λ	n	λ	n
1.009 μ	1.644	1.007 μ	1.481
2.193	1.624	2.066	1.476
2.351	1.621	2.301	1.475
2.430	1.620	2.447	1.474
2.586	1.616	2.593	1.474
2.664	1.615	2.667	1.473
2.742	1.613	2.886	1.472
2.974	1.608	3.032	1.472

From the values given in this table, curves were accurately plotted, by means of which the indices for any intermediate wave length could be obtained.

For a given wave length the refractive index of the extraordinary ray transmitted by the plate whose optic axis made an angle α with the face, was obtained from the formula

$$\frac{1}{n^2} = \frac{1}{n_o^2}\sin^2\alpha + \frac{1}{n_e^2}\cos^2\alpha, \qquad (14)$$

where n_o and n_e are respectively the maximum and minimum indices obtained from the dispersion formulas, and n the desired index. Carvallo's constants for the dispersion formula give indices which agree remarkably well with observed results, and were verified by him in the infra-red as far as $\lambda = 1.98\,\mu$ for the ordinary or maximum index, and to $\lambda = 2.15\,\mu$ for the minimum index.

The method of correction for reflection used supposes a perfect surface of the crystal plate. On account of its softness and marked cleavage, perfect surfaces of Iceland spar are hard to maintain. The tendency for the surfaces to deteriorate is more marked for plates cut in some directions than for others. Before the work was completed, the surfaces of two of the plates used became slightly imperfect as seen by the eye. Although this would not affect the transmission of the longer waves as much as it did the visible rays, yet it would have a tendency to make

the observed transmission too small or the absorption too great. The crystal plate C shows this effect, and to a less extent it is also shown by plate G, as will appear from the results given.

As the plates differed slightly in thickness, a correction for this was applied to the values observed, after they had been corrected for reflection. This correction was computed in accordance with the law of thickness, and was simply equivalent to finding the percentage of transmission for the unit of thickness.

The last correction applied was for the error due to the incomplete polarization. The vibrations transmitted by the polarizer may be resolved into two components, viz., a vibration in the plane of polarization, and one at right angles to it. If the percentage of the intensity of the component vibrating in the plane of polarization be known, the correction may be applied by the following formulas:[1]

$$B_o = a_o(1 - x) + ax,$$

or
$$B_o = a_o + (a - a_o)x, \tag{15}$$

and
$$B = a(1 - x) + a_o x,$$

or
$$B = a - (a - a_o)x, \tag{16}$$

where x is the percentage of the component of the vibrations in the plane of polarization. B_o and B are the observed percentages, after other corrections were made, transmitted by the plate for the ordinary and extraordinary rays respectively. a_o and a are the coefficients of transmission, or the percentage not absorbed, of the ordinary and extraordinary ray.

If P_o and P_e are respectively the actual percentage of the initial intensity of the ordinary and extraordinary rays of the polarizer transmitted,

$$x = \frac{P_o}{P_o + P_e}. \tag{17}$$

But
$$P_o = (1 - r_o)^2 a_o^d,$$

and
$$P_e = (1 - r_e)^2 a_e^d,$$

where d is the thickness of the polarizer in terms of our arbitrary unit of thickness, 3.98 mm., and a_e is the maximum coefficient of transmission of the extraordinary ray.

[1] The Fresnel hypothesis is here followed, viz., that in a beam of plane polarized light the direction of vibration is perpendicular to the plane of polarization.

Neglecting the difference between r_o and r_e we have

$$x = \frac{a_o^4}{a_o^4 + a_e^4} \tag{18}$$

With a plate cut perpendicularly to the optic axis, no matter whether the light is polarized or not, a_o is the coefficient of transmission which will be obtained. This may be seen from equation (15); since for this particular plate the extraordinary and ordinary vibrations become identical, or $a = a_o$. But if x is not zero, the a_e of equation (18) cannot be directly observed. However, it was obtained in the following way. The observed transmission, for the extraordinary ray, of a plate cut with its optic axis parallel to the face was corrected for reflection and thickness. This value gives B in equation (16), which equation, as this is the plate corresponding to maximum transmission, may be written

$$B_e = a_e - (a_e - a_o)x. \tag{19}$$

This value of B_e is a little less than a_e, usually only 1 or 2 per cent. This numerical difference was estimated, and the value obtained for a_e substituted in equation (18), and a value of x obtained. This value was used in (19) to obtain a_e more accurately, and this new value of a_e used in equation (18), and a corrected value of x found, and so on. As x was always a small quantity, and the correction therefore small, it was not necessary to obtain x very accurately. However, the method admitted of a higher degree of accuracy than was used.

Knowing x and a_e, a was obtained for the different plates used, from equations (15) and (16). From these values of the coefficients of transmission the coefficients of absorption were obtained by the relation

$$-m = \log_e a.$$

As has been previously stated, the transmission for the ordinary ray should be the same for all the different directions of propagation. In Table III. are shown values obtained for the transmission of the ordinary ray.

TABLE III.

ORDINARY RAY. $\lambda = 2.43\,\mu$.

Plate.	Observed percentage transmission.	Corrected for reflection.	Corrected for thickness.	Coefficients of transmission.
A	0.410	0.460	0.464	0.452
B	0.416	0.467	0.467	0.458
C	0.419	0.470	0.450	0.445
D	0.407	0.457	0.443	0.439
E	0.399	0.448	0.434	0.430
F	0.393	0.441	0.441	0.440
G	0.404	0.454	0.422	0.422
H	0.405	0.455	0.450	0.450
Mean	0.406	—	—	0.442

As shown in the table, data were taken on all the plates. In the second column are given the percentages actually observed. In the third, the observed values are given after correction has been made for reflection. In the next column these values have been reduced to the values corresponding to the unit thickness. And in the last column are given the values of the coefficients of transmission, which are obtained from the fourth column by applying the correction for polarization.

TABLE IV.

EXTRAORDINARY RAY. $\lambda = 2.28\,\mu$.

Plate.	Observed percentage transmitted.	Corrected for reflection.	Corrected for thickness.	Coefficients of transmission.		Coefficients of absorption.		
				Observed.	Becquerel computed.	Observed.	Mallard computed.	Voigt-Carvallo computed.
A	0.891	0.959	0.960	0.983	—	0.017	—	—
B	0.823	0.892	0.892	0.911	0.921	0.093	0.099	0.081
D	0.664	0.729	0.720	0.729	0.771	0.316	0.301	0.260
E	0.671	0.737	0.728	0.737	0.768	0.305	0.305	0.265
F	0.575	0.638	0.638	0.643	0.659	0.442	0.452	0.419
G	0.518	0.580	0.551	0.551	0.570	0.596	0.572	0.563
H	0.505	0.567	0.563	0.563	0.556	0.574	0.590	0.587
Ordinary ray	0.505	0.567	0.571	0.548	—	0.602	—	—

TABLE V.

EXTRAORDINARY RAY. $\lambda = 2.38\,\mu$.

Plate.	Observed percentage transmitted.	Corrected for reflection.	Corrected for thickness.	Coefficients of transmission.		Coefficients of absorption.		
				Observed.	Becquerel computed.	Observed.	Mallard computed.	Voigt-Carvallo computed.
A	0.891	0.959	0.960	0.979	—	0.021	—	—
B	0.823	0.892	0.892	0.907	0.913	0.098	0.112	0.092
C	0.671	0.733	0.720	0.728	0.791	0.318	0.282	0.241
D	0.662	0.726	0.717	0.725	0.752	0.322	0.335	0.291
E	0.649	0.712	0.702	0.710	0.749	0.342	0.340	0.296
F	0.560	0.622	0.622	0.626	0.632	0.468	0.503	0.467
G	0.492	0.550	0.520	0.520	0.536	0.654	0.636	0.626
H	0.470	0.527	0.523	0.523	0.522	0.648	0.656	0.651
Ordinary ray	—	—	—	[1]0.513	—	0.668	—	—

[1] Obtained from the curve.

TABLE VI.

EXTRAORDINARY RAY. $\lambda = 2.49\,\mu$.

Plate.	Observed percentage transmitted.	Corrected for reflection.	Corrected for thickness.	Coefficients of transmission.		Coefficients of absorption.		
				Observed	Becquerel computed.	Observed	Mallard computed.	Voigt-Carvallo computed.
A	0.882	0.949	0.950	0.962	—	0.039	—	—
B	0.796	0.862	0.862	0.871	0.890	0.138	0.145	0.122
C	0.642	0.702	0.688	0.693	0.756	0.367	0.344	0.295
D	0.626	0.687	0.677	0.682	0.715	0.383	0.405	0.354
E	0.613	0.673	0.662	0.667	0.711	0.405	0.410	0.359
F	0.501	0.556	0.556	0.558	0.583	0.583	0.600	0.559
G	0.450	0.503	0.472	0.472	0.478	0.751	0.754	0.744
H	0.421	0.472	0.467	0.467	0.463	0.761	0.777	0.774
Ordinary ray	0.408	0.458	0.453	0.453	—	0.792	—	—

In Tables IV., V., and VI. are given the values for the variation in the transmission of the extraordinary ray. These are tabulated similarly to the values given in Table III., but with the addition of

the coefficients of absorption, and the theoretical values according to the theories of Becquerel, Mallard, and Voigt-Carvallo. These theoretical values were computed from the observed coefficients corresponding to the maximum and minimum transmission, viz., the coefficients obtained from plate *A* and from the ordinary ray.

In Plates I., II., and III. the observed and theoretical values are shown graphically. The attention of the reader is especially directed to this graphical representation, where conclusions may be more quickly formed than in an inspection of the tabulated data.

In the upper set of curves in Plates I., II., and III. are plotted the observed percentages transmitted and the coefficients of transmission, while the broken curve represents the theoretical values of Becquerel's theory. It is at once evident that the results are not in harmony with the theory of Becquerel. The same conclusion is reached by an inspection of the numerical values given in the tables.

In the lower set of curves in the plates are given the coefficients of absorption. The continuous line is the curve obtained from Mallard's formula, while the broken line is that obtained from the Voigt-Carvallo formula. It will be noticed that the Mallard curve agrees closely with the curve that one would draw for the observation curve, while the Voigt-Carvallo curve lies so far below the Mallard curve as to make the distinction obvious.

The point plotted for the abscissa 44°.4 is in each case the mean of the values obtained from the crystal plates *D* and *E*. On this account, and from the fact that the surfaces of these two plates were almost perfect, greater weight should be given to these values. Although they occur at about the maximum divergence of the Mallard and the Voigt-Carvallo curves, in every case they agree closely with the Mallard value. The observed coefficients plotted for the angle 39°.5 are the values obtained from the crystal plate *C*. This plate is the one that had the poorest surface, and for this reason it always shows a greater absorption. But the variation is not great. The crystal plate *G*, the plate which was cut at an angle of 77° to the optic axis, also showed slight irregularities of its surface.

The Moreau formula has not been compared directly with the

observed results. The variation in the refractive indices of the plates used was so great that his formula as given in equation (7) does not apply. When there is a marked difference in the refractive indices, this formula is unsymmetrical, while the data given here show a symmetrical relation.

It is also apparent from these results that the error due to incomplete polarization was very small. In Table III., if any error of this kind were present, the observed coefficients of transmission of the crystal plates *A* and *B* would be larger than the coefficients obtained from the plates *F*, *G*, and *H*. For in the case of the plate *A*, if the rays were not entirely plane polarized, the plate would transmit both an ordinary and an extraordinary ray. As the transmission by this plate of the extraordinary ray is almost perfect, the result would be an increase in the transmission observed. On the other hand, the transmission of the extraordinary ray by the plates *G* and *H* is nearly the same as that of the ordinary ray, and the presence of the former would not change to a noticeable extent the transmission observed.

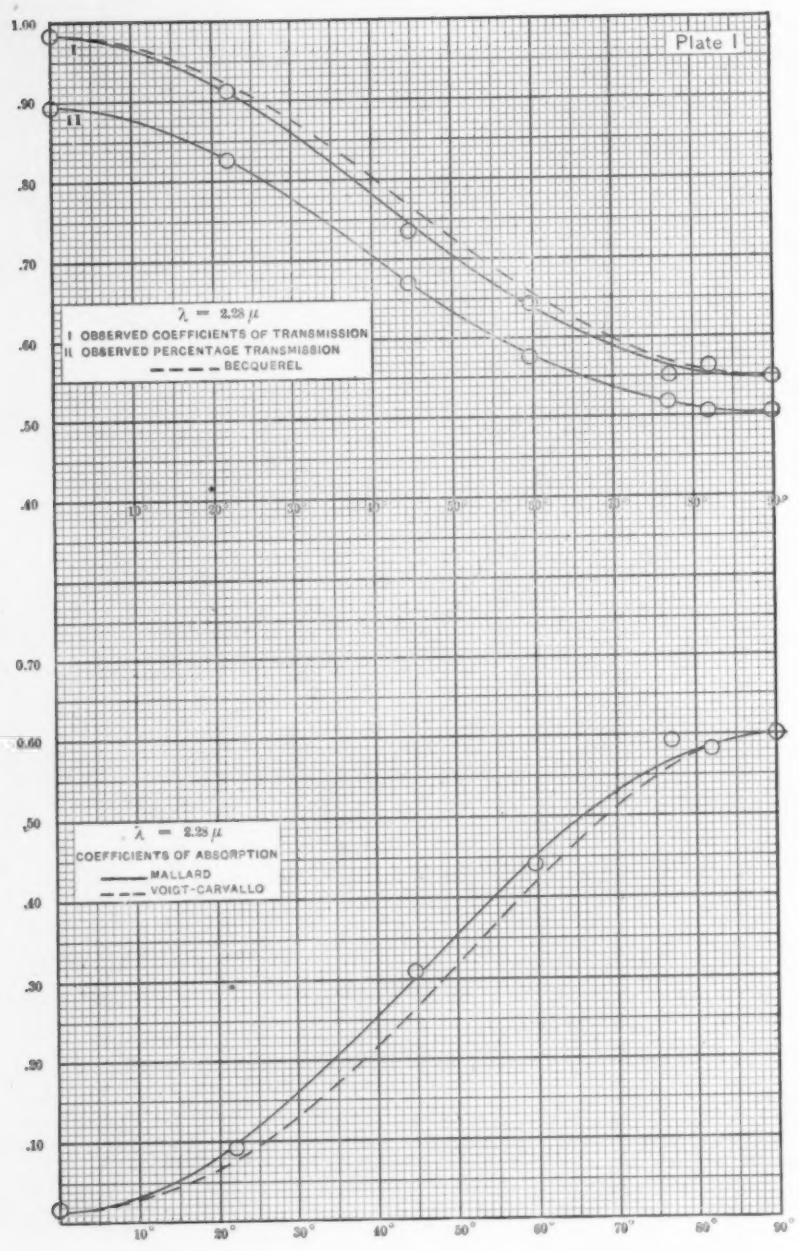
Again, it is seen that the maximum coefficients of transmission obtained for the extraordinary ray are only slightly less than that corresponding to a perfect transmission, while the minimum, or that of the ordinary ray, is relatively small. If any considerable error, due to incomplete polarization, was present, the observed values for the maximum transmission would be smaller than those actually obtained. That there is a slight absorption of the extraordinary ray corresponding to the maximum transmission, follows from the observations of Professor Merritt. He found that a slight absorption of the extraordinary ray that corresponds to the maximum transmission was present in this region, and that the absorption increases slightly with the wave length. This is confirmed by the data obtained by the writer.

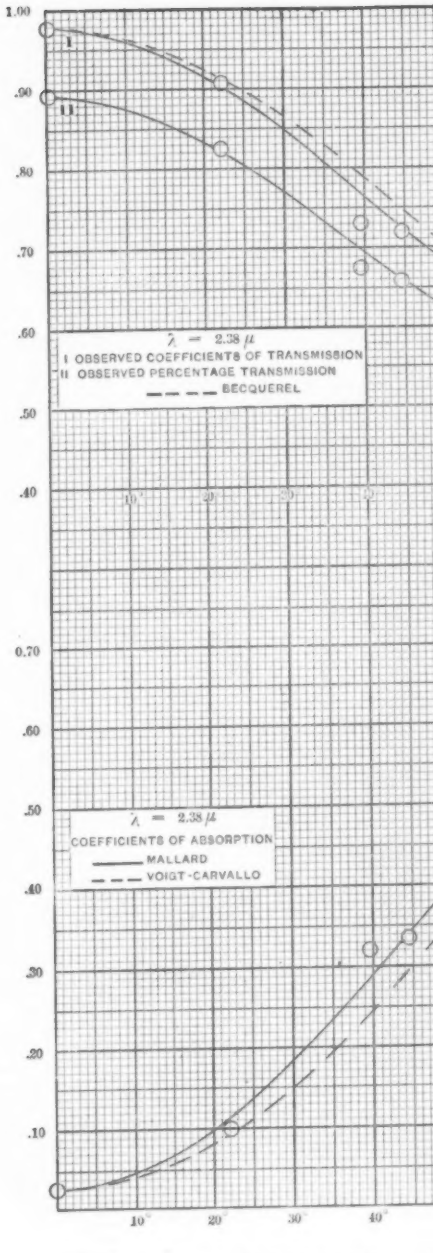
The crystal plates *D* and *E* were cut with faces respectively parallel to the negative and positive unit rhombohedron. Although the angles made with the optic axis by the faces of two such plates are equal, other crystalline properties are different, the face of one plate being parallel to one of the cleavage planes, while the face of the other was equally inclined to two of the cleavage planes.

These plates were obtained for the purpose of testing whether the direction of the cleavage plane affected the absorption. The plate *E* shows, in Tables V. and VI., a slightly greater absorption than plate *D*. This is explained by the fact that the face of plate *D* was not quite parallel to the negative rhombohedron, and the angle made by the face with the optic axis was a little smaller than the angle made by the plate *E*. But in Table IV. the errors of observation have concealed even this effect. The conclusion is therefore drawn that if the direction of cleavage has any effect on the absorption, the effect is less than the errors of observation.

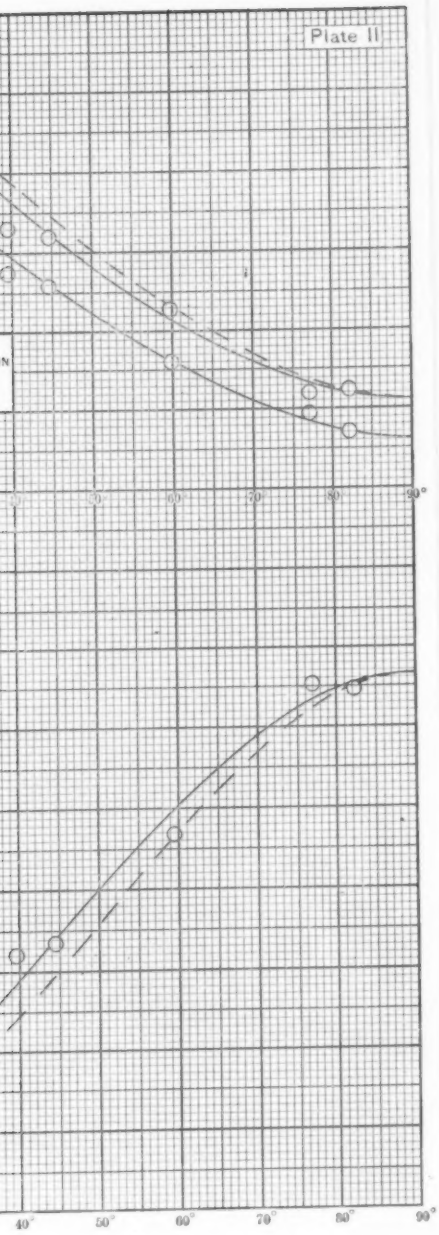
This work was begun before the papers of Carvallo and Camichel had been published. After the publication of the work of Carvallo, the writer was more inclined to favor the Voigt or Carvallo theory than that of Mallard. But from the preceding results the conclusion must be drawn that the theory of Mallard agrees with experiment more nearly than any of the others.

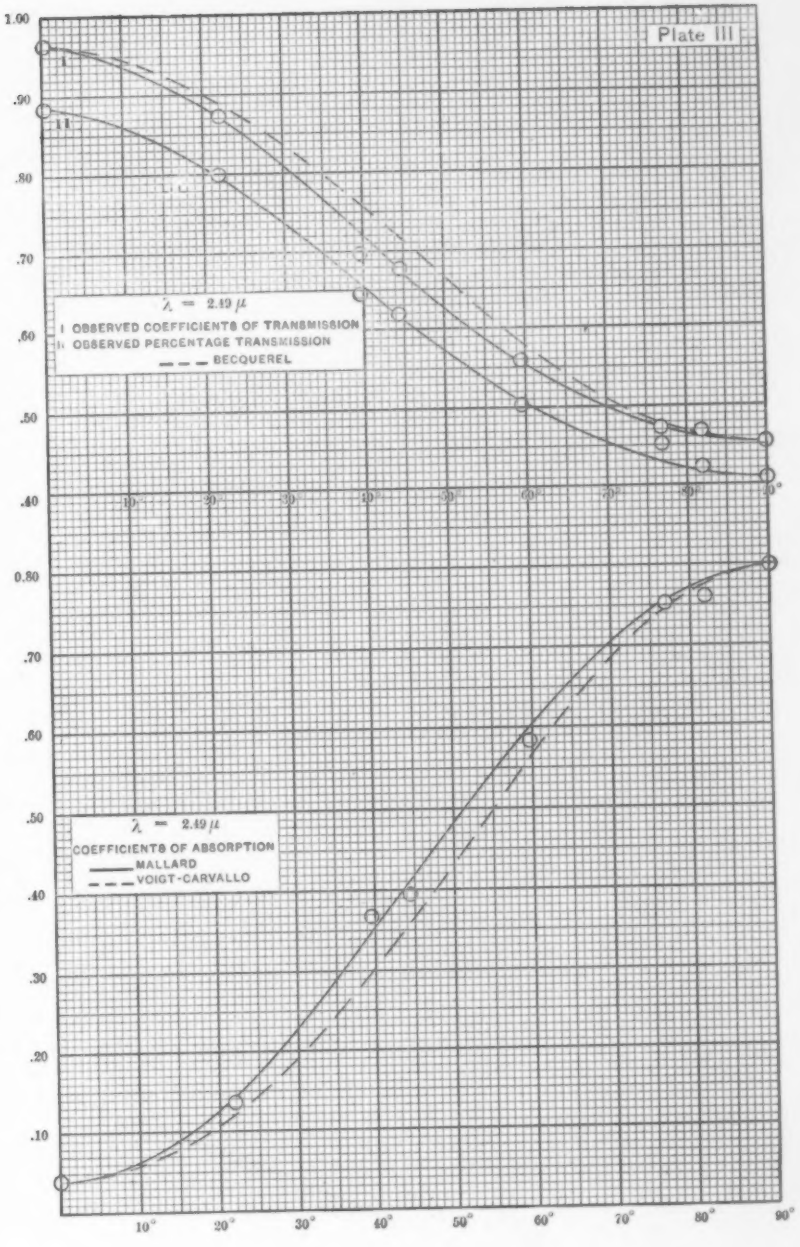
PHYSICAL LABORATORY, CORNELL UNIVERSITY.





STEWART: ABSORPTION I





TION IN UNIAXAL CRYSTALS.

ON THE CONVERSION OF ELECTRIC ENERGY IN DIELECTRICS. I.

By Richard Threlfall.

IN the *Journal de Physique* for 1892, p. 5, is a very interesting paper by M. Bouty on the properties of mica. It appears that under certain circumstances the residual charge of M. Bouty's mica condensers may fall to about $\frac{1}{80}$ or less of the original charge.

In September, 1892, I was experimenting with films of exceptionally well purified sulphur, and found, conformably with other observers, that the residual charge of crystalline sulphur, regarded as a dielectric, is remarkably small. This led to a repetition of some of M. Bouty's work on mica, for I desired to compare the behavior of mica and sulphur under exactly corresponding conditions. In the course of this comparison it appeared that *prolonged* drying over phosphorus pentoxide materially reduced the apparent residual charge both with mica and sulphur. As this observation seemed to throw some doubt on the sufficiency of previous work in this direction, the literature of the subject was closely examined, with the result that, with the exception of an investigation by Rowland and Nichols (*Phil. Mag.*, 1881, Vol. 12, p. 418), the distinction between "residual charge" and "creeping surface charge" does not appear to have sufficiently engaged the attention of observers. This led to a desire to examine the general properties of dielectrics in a comprehensive manner; but it was not until September, 1895, that I was able to make any serious attack upon the subject. Meanwhile, I became acquainted (through the technical journals) with the fact that when ordinary mica or paraffin paper condensers are used with alternating currents, great heating generally occurs. This heating was usually ascribed to "dielectric hysteresis," although no definition of the term had been given and no experiments of an unexceptionable kind on the subject had been published.

Lord Kelvin expressed an opinion, in the course of a discussion at a meeting of the Society of Electrical Engineers some years ago, that the cause of condenser heating was most likely to be found in the occurrence of internal discharges rendered possible by the presence of air and moisture accompanying the dielectric employed.

Mr. Swinburn, Mr. Tesla, and perhaps others have given attention to the production of condensers on the commercial scale, though with only

a certain measure of success, so far as the tests at my disposal have enabled me to ascertain. From the experiments described above, however, I knew that success must depend not on the partial removal of air and water, but on the absolute removal of these substances, and by experimenting in this direction I have been able to make condensers in which the heating by alternating currents is practically negligible, although the dielectric used was merely paper saturated with a mixture of vaseline and hard paraffin. As I have described the mode of preparation of these condensers at full length in a book on Laboratory Arts, I will not refer to it here, but will merely offer the following data for comparison.

A certain condenser of a capacity of 0.123 microfarads was tested at an alternating potential difference of 3000 volts and with a frequency of 60 periods per second. This condenser when packed in a box and thermally insulated was found to rise in temperature by an amount certainly less than one fifth of a degree centigrade per hour. Compare this with some tests by Messrs. Bedell, Ballantyne, and Williamson (PHYSICAL REVIEW, I., p. 95, Sept.–Oct., 1893) on a waxed paper condenser. This condenser was tested at a potential difference of 500 volts and with a frequency of 160. The loss was ascertained to be from 2 to 3 per cent of the power transmitted; and the rise of temperature, under circumstances similar to my own tests, was at the rate of about 1° C. per hour. In order to see what this comparison means, it is more convenient to consider that energy moves first in and then out of the condenser, than to begin by writing down the activity equation, which, in the case of no condenser loss, is equivalent to saying that no energy is supplied. Looking at it in the way I propose, it appears that the energy supply in the case I tested was, per unit capacity, about thirteen times greater than in the case treated by Messrs. Bedell, Ballantyne, and Williamson. In making a comparison, the actual capacity is, roughly, immaterial, since the volume of condenser matter to be heated and the capacity of the condenser increase together. Assuming that the specific heat of the condenser materials and the heat insulation was the same in both cases, we find that the percentage losses are in the ratio of about 1 to 60 — or the condenser I tested was only losing energy to an extent of clearly less than $\frac{3}{60}$ per cent = 0.05 per cent of the energy supplied. I mention this chiefly as an illustration of the immense effect which small quantities of air or moisture have on the apparent behavior of dielectrics. Of course it is evident that no information as to the purely dielectric action of dielectrics as opposed to their conducting action can be drawn from such experiments, unless it be known with certainty that the conduction losses may be neglected — and this, in general, it is next door to impossible to ascertain. I need only refer to the experiments of Koller (*Wien. Bericht*, 98, II., 201, 1889) on the specific resist-

ance of oils, for a justification of this statement, for this investigation shows how the specific resistance is a matter of previous history, voltage, etc., a conclusion entirely in accordance with my own experience where the resistance of insulators is in question.

The question which I desired to approach experimentally in 1893 was this: Is it true that when surface action is properly guarded against and an elementary volume of a dielectric is carried round a cycle of electric polarization, that some of the energy of electrification is converted into heat, or, at all events, disappears? Assuming that this question is answered in the affirmative (it is), the further question arises as to whether this loss can be explained on the lines suggested by Maxwell (*Electricity and Magnetism*, § 328) and more recently elaborated by Hess (*Journal de Physique*, 3d series, Vol. 2, 145, April, 1893). Although I cannot claim to have finally answered this question, I think the experiments which will be described lead to the conclusion that Maxwell's explanation, though a possible one, is not sufficient in all cases. A third question arose during the investigation, viz., as to whether dielectric loss under prescribed circumstances is a fixed physical property of the dielectric, or whether it is of the nature of an accidental circumstance depending on such minute peculiarities that we are unable in any particular case to predict what the loss will be. Does it in fact resemble the problem of the prediction of hysteresis loss in an untested specimen of iron? This question is finally resolved in the sense that the dielectric loss, though perfectly determinate in any given specimen, is exceedingly variable in passing from one specimen to another, even though every care be taken to secure uniformity. Thus, the loss in the outer layers of a rod of ebonite is different to the loss at the center (per unit volume, of course), and two spheres of resin cast at the same time from the same melting of resin may behave quite differently.

The investigation has been extended to very rapidly varying electric forces, produced by the use of oscillating discharges, in the hope that some light may be thrown on the curious opacity of certain dielectrics — black waxes for instance — but with the unexpected result that dielectric losses become either zero, or, at least, get too small for detection at frequencies of the order of 10^7 per second.

A large number of subsidiary matters, such as the absence of losses in liquid dielectrics, were also investigated.

Before giving an account of my own experiments, it is necessary to deal briefly with the work of other observers in the same field. A detailed consideration would be out of place here, for in the *Electrician* for November 1, 1895, there is an elaborate discussion of the whole position by P. Gasnier. I may as well say here, however, that I consider that in this matter there has been too great a tendency to take things for granted. M. Gasnier

assumes the fact of a dielectric hysteresis analogous to magnetic hysteresis, and discusses the question as to whether this hysteresis is "static" or "viscous." The following brief criticism of the then available evidence will, I think, show that this discussion was premature, but my own experiments have, I think, settled the question in favor of the latter hypothesis, at least in certain cases, in the event of Maxwell's suggestion being regarded as insufficient, but not otherwise.

Steinmetz, *Electrotech. Zeitschr.*, 1892, April 29. This paper has been often quoted as proving the existence of dielectric hysteresis. It does no such thing. Mr. Steinmetz measured the phase angle between current and potential difference in a paraffin paper condenser circuit by a dynamometric method, and deduced the condenser losses. The insulation resistance of the condenser was determined by means of a steady electromotive force of the order of the alternating potential difference. The results thus obtained are, of course, quite fallacious on the grounds of uncertainty of resistance; independently of this, however, the greater portion of the loss was probably due to creeping and air discharges.

A very persistent worker in this field has been Riccardo Arno.[1] This investigator made use of an original method of great ingenuity and convenience. An electrostatic field is caused to rotate by the device of combining two mutually perpendicular, harmonically varying electric fields. The harmonically varying potential differences with the necessary phase difference are obtained by placing a small condenser in series with a water resistance and the secondary of a transformer. The general arrangement will be clear from a reference to the diagram which accompanies the latter portion of this paper. The direction of rotation of the field can be reversed by reversing one pair of connections. The armatures bounding the rotating field are formed of portions of a cylindrical surface fashioned in metal and supported on an insulating base.

A small hollow cylinder of ebonite was found to rotate when placed in the space between the armature surfaces.

Arno takes the supposed fact of dielectric hysteresis for granted, and refers to Steinmetz as having established it.

In the second paper on this subject Arno suspends a small hollow ebonite cylinder by a bifilar suspension in the rotating field, and thus measures the moment of the forces brought to bear upon it. This immediately gives the loss of energy per revolution — or per cycle of electric polarization. It is important to note that M. Arno dried his cylinder by a cake of calcium

[1] Riccardo Arno, Accad. dei Linc., Oct., 1892; Elec. Review, Jan. 20, 1893; Accad. dei Linc., April, 1893; Electrician, 31. 201. Also Electrician, 32. 222; Electrician, 33. 210; Electrician, Jan., 1895; Electrician, May 15, 1896.

chloride which was inclosed in the apparatus. The potential difference at the terminals of the transformer (an induction coil) varied from .95 to 3·33 electrostatic units. The results of the measurements were represented by a formula of the form

$$D = CV^{1.6},$$

where D is the angular deflection of the cylinder, C a constant, and V the potential difference employed.

In the third paper the range of electric force is considerably extended, and a long series of numerical results are given as deduced from experiments on paraffined cardboard cylinders: the frequency of field rotation was in all cases 40 per second. Arno thinks that the similarity of his results to those obtained by Ewing and Miss Klaassen in the case of iron lead to the conclusion that he is dealing with "static" hysteresis.

In the fourth paper the frequency of rotation as well as the potential difference is varied, the material still being paraffined cardboard, with the result that the energy waste is found to decrease as the speed of rotation increases — both at low and high potential differences. This causes M. Arno to reconsider the conclusion come to in his former papers.

Arno's conclusions, as distinguished from his experiments, have been sharply criticized by M. Hess in *L'Éclairage Électrique*, Vol. 7, p. 450 (June, 1896). M. Hess very properly takes exception to the use of such heterogeneous material as paraffined cardboard, and shows that Maxwell's suggestion is quite sufficient to explain the results obtained without any reference to the doctrine of hysteresis at all.

I have made large use of M. Arno's method, though in a modified form, for it has many advantages; and I therefore reserve for the present my own remarks on the subject. I regret to say that the *Rendiconti dei Lincei* is not available in Sydney, and I therefore had to trust to translations and abstracts for this brief summary: I can only hope that I have not been misled by them, and have done M. Arno no injustice.

Perhaps the most interesting work which has yet appeared on the subject of dielectric polarization is contained in various notes sent to the *Comptes Rendus* by M. Janet.

In *Comptes Rendus*, 115, 875 (1892), Janet explains his method of experimenting, which consists in breaking the circuit of a condenser, induction coil, and dead resistance, and the instantaneous measurement of certain potential differences by means of "M. Mouton's Disjoncteur." In this paper the theory of the apparatus is given and the oscillations of the condenser discharge are traced out.

In *Comptes Rendus*, 115, p. 1286, the apparatus is applied to determine the inductance of the circuit, and the result is checked and found

to be correct by a measurement of the same quantity by Lord Rayleigh's method.

The third paper, to which the others must be regarded as preliminary, is in *Comptes Rendus*, 116, 373 (1893), and is devoted to the study of a certain mica condenser. The ratio of the charge on the condenser to the potential difference of its terminals, forms the subject of inquiry, and it is found that for given values of rapidly changing potential difference the charge is *less* with rising than with falling values. If charge and potential difference are plotted round a cycle, a closed curve or hysteresis diagram is obtained. It is obvious, however, that this is just the effect to be expected if we allow the possibility of the creeping of the charge of the condenser armatures over the imperfectly dried mica surface. But this possibility, I think, has been established by the repetition of M. Bouty's experiment, described at the commencement of this paper. In addition to this, I consider that some criticism of the results obtained by M. Janet may be applied on other grounds; but as this does not materially affect the position, it may be omitted here.

Some rather interesting experiments are described by Benischke (*Wien. Sitzber.*, 102, Part II., p. 1345) on the heating of condensers. A bolometric method was employed, with the result that with glass, mica, and ebonite, some heating was observed; with resin and paraffin, however, no heating was noticed. The author's conclusion is that the heating is due to Joulean wastes and mechanical effects. Benischke refers to several other workers at the subject, but on reference I found that their researches have no bearing on the present inquiry.

In the *Comptes Rendus* for May, 1893, M. Borel described a new phenomenon under the title of "Phénomènes dynamiques dus à l'électrisation residuelle des diélectriques." A disk of some dielectric material is suspended between two conducting plates so that the plane of the disk is horizontal, and in the plane of the lines of electrostatic force. The plates are supplied with an alternating potential difference by commutating the current from a Toepler machine. It is found that when a glass rod is held near to one edge of the suspended disk a sort of repulsion is observed which leads to a rotation of the disk. The drift of M. Borel's explanation will be gathered from the title of his paper. This explanation is, however, merely speculative, and though no doubt M. Borel's experiments deserve careful repetition and variation, I am unable to see that they necessarily prove the fact of "hysteresis."

A large number of experimenters have drawn attention to the very variable values of the dielectric constant obtained when very rapid methods are compared with slower ones. M. Blondlot, Professor J. J. Thomson, Boltzmann, and Northrup may be mentioned in this connec-

tion.[1] The general result appears to be that when the dielectric constant is determined by very rapid methods (oscillatory discharges) it tends towards a smaller value than that arrived at by charges of longer duration.

Perhaps Mr. Northrup's paper on this subject places the matter in the clearest light (*Phil. Mag.*, Vol. 39, Jan., 1895).

In the foregoing summary I have confined myself to work which I have seen quoted as leading to the probability of dielectric hysteresis. The list of investigators of the charge of condensers might be immensely extended, and hardly any observer in this field has failed to note some matter requiring further explanation. With regard to dielectric losses, however, there are no experiments, so far as I know, which shut out the possibility of explanation by means either of conduction or of creeping, or of both.

During the progress of an investigation of the properties of sulphur made some years ago, I was much struck by the huge differences assigned to the specific inductive capacity of that substance by different observers; differences which I satisfied myself were really quite outside the range of experimental error, and which I, at the time, attributed to want of definition in the specification of the chemical state of the sulphur employed. In the course of the investigation referred to, I made a very large number of determinations of the dielectric constant of sulphur under varying conditions, and encountered variations in the behavior of even well-specified samples, which, if not entirely outside the range of experimental error, still left in my mind the impression that the specific inductive capacity of apparently similar samples was rarely exactly the same.

Perhaps the most startling inconsistency occurs in the case of muscovite (mica), for which M. Bouty found a well-defined specific inductive capacity of about 8 (*Journal de Physique*, I., 1892, p. 5), while M. Klemenčič by equally careful experiments obtained the value 6 for the same constant. With the help of Wiedemann's *Sachregister*, 1824–1877, and the Beiblätter of Wiedemann's *Annalen* since that date, I have found a large number of observations on specific inductive capacity, many of which, of course, are largely affected by imperfect methods of measurement, but which, nevertheless, give an impression that the constant depends very much on the sample of material selected for observation.

Experimental Work. — In September, 1895, I set up, with the assistance of Mr. Pepper, a copy of Arno's apparatus, with the object of finding out whether it was suitable for qualitative work on the subject of dielectric losses. There is a considerable difficulty in making a suitable high non-

[1] J. J. Thomson, Phil. Proc. R. S., 46, 292 (1889); Blondlot, Comptes Rendus, 112, 1058 (1891); Boltzmann, Gordon's Electricity and Magnetism.

inductive resistance, and like Arno, I was driven back upon the use of a column of water. This has the disadvantage of being appreciably heated by the currents employed; for the main condenser must have considerable capacity to swamp the effect of the armatures of the rotating field apparatus. No doubt the variability in resistance which gave rise to the trouble might have been got over, but I soon found that the arrangement was unsuitable in another particular.

Consideration of the diagram will render it obvious that the field produced within the armatures can only be very roughly "circularly polarized." The leakage of induction between the quadrants placed nearest to one another varies considerably during each cycle, with the result that the harmonically compounding fields, though doubtless of the same mean intensity, are not so at any prescribed moment. By this, of course, I mean that there is a variation of intensity quite large in comparison with the sinusoidal variation postulated by the theory. The polarization must, therefore, be elliptic, and the ellipticity must be variable during each cycle. By roughly drawing the field, under certain arbitrary conditions, I was able to see that in some cases and at points in the field not far from the center, the polarization must be almost *plane*, and this I verified by suspending a mica needle, and observing its behavior at the points in question. As had been anticipated, the needle refused to rotate when placed in certain positions, and vibrated instead about a well-marked direction. It was clear, therefore, that the intensity of the field could not be exactly determined in absolute measure, even at a specified epoch, while if the whole cycle were considered, the variations of strength were so great as to render any exact quantitative conclusion entirely ridiculous.

On the other hand, the method has such conspicuous merits that I considered it advisable to adhere to it as far as possible. In the first place, the quantity sought, viz., the loss of energy, forms the direct subject of the measurement, and that measurement is, moreover, of a remarkably precise and simple kind, viz., the torsion of a quartz fiber. The material employed can also be perfectly and continuously dried; and is used in a form very favorable for the observation of its physical condition. Most important of all, the slope of the potential gradient is very gradual all over the surface of the sample; a condition of affairs exceedingly favorable to the reduction of creeping charges, the uncertain and deceptive action of which I was particularly anxious to avoid. A further check on this source of error can also be easily attained by coating the samples of dielectric superficially with paraffin, varnish, etc.

The modification of Arno's method which I was led to adopt was (1) the substitution of a mechanically rotated uniform field for a field produced by the composition of harmonic motions, and (2) the presentation of the

SECTIONAL ELEVATION OF ROTATING CELL.

C. Spring for making electric contact with myograph.
P. Contact pin for ditto.
H. Driving belt to motor.
K. Driving belt to speed indicator.
DD. Brushes making contact with Wimshurst machine.
EEEE. Contact rings.
S. Thin glass beaker for holding drying material.
Q. Drying material.
V. Vanes.
The darker lines of the guard tube and mirror box indicate tinfoil screens.
RR. Movable horizontal screens for protecting mirror and vane against direct electrification.
(The full length of the vertical rod supporting the suspension is not shown.)

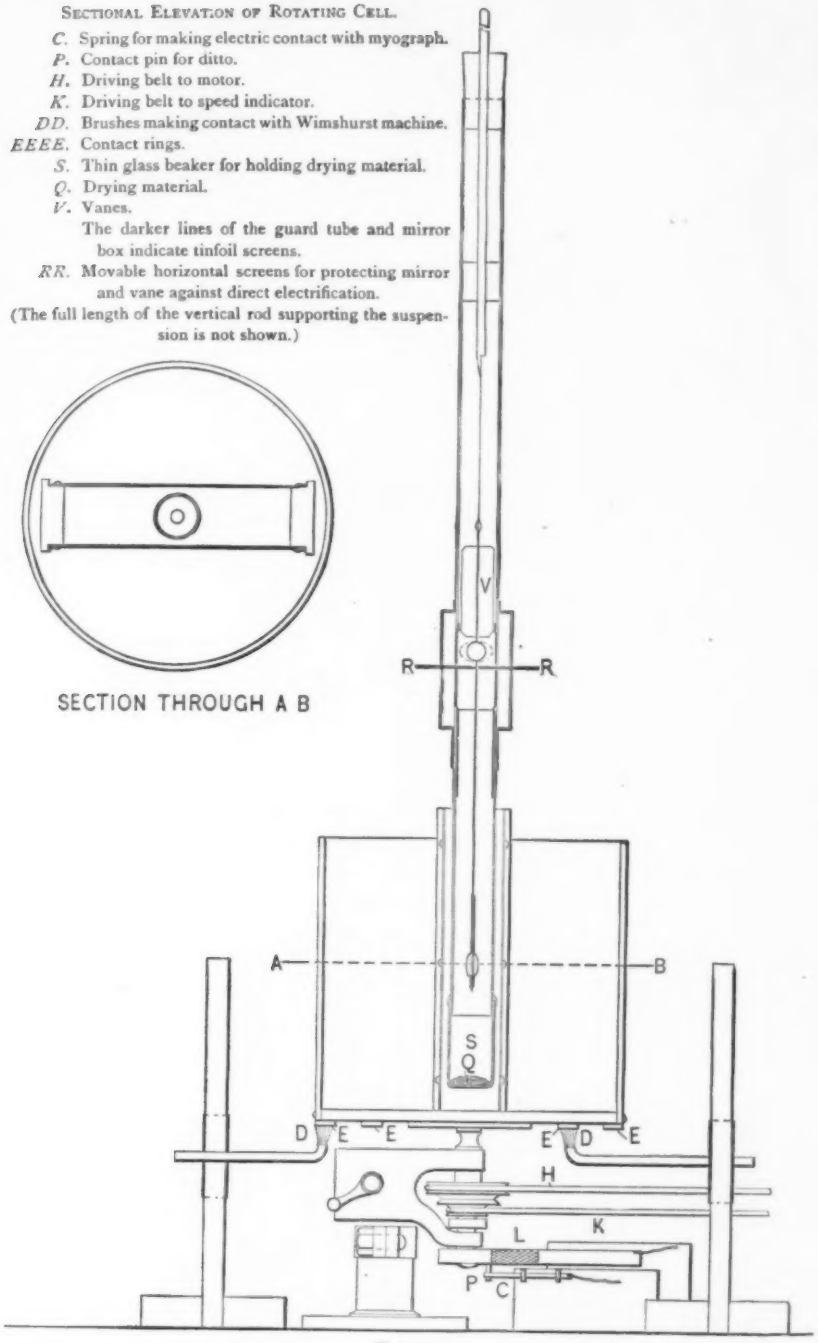


Fig. 1.

substance to be examined in the form of an ellipsoid, which permits us to calculate the actual field to which it is exposed, and hence to reduce the results of observation to absolute electrostatic measure.

The alterations proposed were first thoroughly tested by means of an apparatus chiefly constructed of wood; and the experience obtained was then applied to the construction of the permanent apparatus, which is shown partly in elevation and partly in section in Fig. 1. The base of the rotating part is formed of the head stock of a watchmaker's lathe, suitably supported. This carries the revolving cell, consisting of two zinc plates with suitable stiffening and wire edges, mounted on a disk of ebonite and surrounded by a case of varnished paper which rotates with the cell and prevents it acting too much as a fan when it is driven round at a high rate of speed. The drawing is to scale, but the following dimensions may be quoted. The zinc plates were 3.8 cm. apart. The width of each plate was 15.8 cm., and the depth, *i.e.* the dimension parallel to the axis of rotation, was 17.8 cm. The two zinc plates were of rolled zinc and were about 1 mm. thick; they were held in position by milled ebonite end pieces; and these were firmly screwed to the face plate of the lathe. The diameter of the varnished paper cylinder surrounding the whole was 19.6 cm., and its thickness was 15 cm. Paper was chosen for this purpose, as when wrapped upon a cylindrical former it makes a light but strong protection and can readily be fashioned so as to have the exact size required. The whole of the rotating parts were most carefully balanced — a precaution absolutely necessary. The driving cone of the apparatus was belted to a shunt electric motor, driven from a constant potential circuit, and was governed by the Foucault current resistance which a large brass disk, mounted on the motor shaft, experienced when exposed to the action of a large horseshoe electromagnet with poles at equal distances from the axis of rotation. The electromagnet was provided with current through a variable resistance, and could also be moved with respect to the rotating disk. This method of governing is the precise electrical analogue of the method used by Lord Rayleigh to govern a water motor in his determination of the absolute value of a resistance by the British Association and Lorenz methods. The fanlike action of the cell itself also assists the governing.

The second belt from the driving cone of the cell passed to a speed indicator of novel but simple form. My object was to employ a speed indicator of such a kind that very little power would be required to work it, and which would work over a large range of speed. The advantage of using a speed indicator which required but little power is that it can be driven by a belt without fear of the indications being affected by the slipping of the belt. The speed indicator really consisted of two parts, one

arranged as an inspectional instrument to afford information as to the constancy of the speed, and the latter designed to afford an exact measurement of the speed at any specified instant. The inspectional instrument consisted of a minute fan revolving in a brass case and measuring about 5 cm. in diameter over all. The air pressure produced by this fan was observed in the following way — it was designedly too small for a hydrostatic gauge to be of any use. A wide piece of brass tubing, 7 cm. diameter and 8 cm. long, was closed by means of a cork at one end and was provided at the other with a diaphragm of fine rubber tissue very evenly stretched. The tube was placed with the plane of the tissue vertical, and at one end of the horizontal diameter of the face a galvanometer mirror was fastened. The position of the mirror was observed by a lamp and scale. The air in the brass tube was put in communication with the fan by means of rubber tubing, so that when the fan rotated the air pressure produced was communicated to the diaphragm and caused it to bulge outward, thus slightly rotating the mirror.

The apparatus for registering the speed of rotation — or calibrating the speed indication — consisted merely of a contact arrangement on the vertical shaft, below the driving cone of the lathe head stock. The contacts were made once every revolution and worked the electromagnetic arrangements of a myograph in an adjoining room. The general practice was to calibrate the speed indicator by the myograph so that a rough idea of the frequency of rotation could be obtained, and then this speed being kept constant it could be measured exactly, if necessary, at any convenient moment.

The arrangement worked perfectly from the start, almost any desired degree of sensitiveness being attained by adjusting a leak from the brass tube; and by a pinch clamp working on the rubber tube connecting the indicator with the fan. It takes almost as long to describe the apparatus as to make it. Mine was made during one morning.

The Suspended Apparatus. — A brass tube about 2.5 cm. diameter was soldered to a strong brass bar, so that when the bar was horizontal the tube was vertical. The heavy slide rest of a lathe was placed on a wall bracket immediately above the rotating apparatus (which was clamped to a slate table), and the brass bar was carried by the slide rest. Everything was exceedingly strong, heavy, and rigid, and this is necessary. The brass tube carried the suspended ellipsoid of dielectric matter, and was furnished in the usual way for a quartz suspension. The tube terminated in a cylindrical wooden box in which a window was cut. Below the box and coaxial with the brass tube was a glass tube of thin, flint glass, carefully dried and varnished. This tube ran into a shallow, thin glass beaker containing phosphorus pentoxide, and was cemented to it by soft wax.

The slide rest served as a means of adjusting the suspension tube concentric with the axis of rotation of the rotating plate system, and being itself supported on three screws, perfect leveling could also be obtained.

The thick lines in the diagram which bound the upper part of the glass tube both outside and in, indicate the tin-foil screening which it was found necessary to solder on to the brass tube — itself carefully earthed. Two brass screens were also introduced just below the suspended mirror, for, of course, perfect screening of the mirror is essential. These side screens could be taken in and out as required for the purpose of raising or lowering the suspended dielectric.

The Dielectric and its Mounting. — Samples of dielectric were prepared in various ways. Ellipsoids of various shape and size were employed (including spheres), the largest of which, actually employed for observation, had a horizontal diameter (when mounted) of about 1 cm. The inner diameter of the glass tube was 2.504 cm., and the outer diameter 2.750 cm. It will be shown further on that the field inside such a tube, arranged in an initially uniform field, is itself uniform. When the dielectric is introduced, the field becomes non-uniform in the space outside the dielectric, and the ellipsoid is in unstable equilibrium when suspended coaxially with the tube. For this reason it is desirable, from the point of view of convenience of observation, to keep the diameter of the ellipsoid down to about 1 cm. This conclusion was the result of many trials.

It was a question as to whether the suspension of the ellipsoid should be conducting and earthed, or as perfectly non-conducting as possible. Owing to the convenience of using quartz threads and the inconvenience of silvering them, the latter alternative was adopted, but it led to much trouble. It was not until the ellipsoid was carried on a quartz needle that electrification of the mirror and vane was got rid of. Solid dielectrics, such as ebonite, were prepared by turning on a lathe, and were correctly drilled to receive the quartz needle. Other dielectrics were cast in a sort of zinc bullet mold around a short length of filamentary glass tube thrust through the mold by means of properly bored holes. These narrow tubes were prepared just to admit of the passage of the quartz suspension needle, and formed an easy and accurate mode of suspending the ellipsoids in position. When an ellipsoid had been mounted the quartz needle was heated by a minute Bunsen flame so as to make sure of its insulation, and the ellipsoid was diselectrified by the flame, after which the suspended apparatus was carefully lowered into the suspension tube, which was leveled to center it, and the whole was then adjusted to the rotating cell by means of the screws of the slide rest. A little soft wax made the affair quite air tight, and gave the phosphorus pentoxide an opportunity to exert its proper influence.

As the weight of the suspended parts was not of importance, a substan-

tial glass mirror, carefully figured to a radius of about 1 meter, was employed to indicate deflections, and this was surmounted by a mica vane nearly filling the tube. The semi-transparent scale was of celluloid bent to a radius of 96 cm. — the invariable distance of mirror from scale. Some of the earlier observations were taken on a straight scale and reduced to angles by a graphic method.

Electrification of the Rotating Cell. — As the result of a number of trials, one plate of the cell system was kept to earth, the other being charged by an electrical machine. The plates were kept in contact with earth and the source, respectively, by means of brushes bearing on rings beneath the ebonite disk which formed the base of the cell.

An eight-plate Wimshurst machine, driven by a constant potential motor, was used as a source of electrification. A battery of Leydens was employed to steady its action. The potential difference of the plates was measured by a Kelvin electrostatic voltmeter, whose readings were found to be self consistent, but the absolute value of which was not tested.

The potential difference was kept constant by interposing a short circuit consisting of a pile of vulcanized fiber plates and oiled paper. These were mounted under a screw on an iron stand, and an assistant kept the voltmeter steady by working the screw. A good many plans were tried, but this was found to be the most convenient one.

Sources of Error. — The most important and persistent source of error was found to be a shifting of the zero on the dynamometer scale. At first this was large, but careful screening of the mirror, etc., by tin foil, and the introduction of the horizontal screens under the mirror, immensely reduced it. The ebonite used in the construction of the cell itself also tended to become electrified, and produced shifting. This was prevented from becoming serious by wiping the ebonite ends of the cell now and then with a rag moistened with kerosene. The ebonite base between the conducting rings also became electrified, but all such effects were removed by a thick coating of tin foil well earthed, which lined the whole of the base of the cell and the inside of the paper guard cylinder. This involved raising the zinc plates about 1.5 cm. from the ebonite base, the space being filled in with mica to reduce the fan action. The electrification of the glass guard tube at first gave trouble, but this was got over by never applying the potential difference until the cell was in rapid rotation. The remaining cause of error, viz., the electrification of the dielectric ellipsoid itself, was never entirely got over. It depended on the material of the ellipsoid, and some materials, *e.g.* resin, never gave really satisfactory results from this cause. It was not till near the close of the observations that it was certainly discovered that the shifting zero was dependent on the nature of the ellipsoid. With glass, sulphur, and paraffin I have had samples giving a

zero absolutely steady, so far as I could discover, *i.e.* the light spot did not move by more than 0.1 mm. on the scale. At an early stage it was noted that this shift of zero was a phenomenon requiring time, and it was also happily independent of the direction of rotation of the cell: it depended merely on the potential difference and the length of time this had been applied.

An exhaustive series of experiments was made in order to discover the best mode of observing the deflections, a certain ebonite sphere which exhibited considerable "creeping of the zero" being selected as the subject of examination. A series of measurements was made by electrifying the plates to varying potential differences, observing the rotation of the sphere, and then connecting the plates and observing the zero. This series of observations was compared with a set in which the deflections and potential differences were simply read off with a cell rotating alternately in one and the other direction. The differences only amounted to one or two per cent, and the curves expressing the relation of the logarithms of the deflections of the sphere and the logarithms of the potential difference were indistinguishable. This being established, the latter mode of observation was invariably adopted, subject to the following precautions: The zero reading at the commencement of each set of observations was made identical by diselectrifying the ebonite end pieces, and waiting, if necessary, for the dielectrics under examination to lose their charges. Observations were then made after equal lapses of times at the potential differences agreed upon, first with the cell rotating in one direction, and then with the direction of motion reversed. The observations thus obtained were found to be perfectly consistent on repetition, and even after the material had been dismounted and put back again.

Effect of the Position of the Sphere between the Plates. — The guard tube was always set so as to be coaxial with the rotating system, but experiments were made to test the effect of a sensible want of centering. It was found that when the centering was imperfect, the zero changes became more troublesome, but that only a slight effect was produced on the mean deflections.

Raising and lowering the dielectric in the guard tube had an inappreciable effect even when the range of motion approached a centimeter in either direction. As the cell had been designed to give a uniform field over a range of some centimeters, within the limits of about 1 per cent, this was only to be expected. I was unable to make a rigorous investigation owing to the finite width of the plates, but considerations of symmetry show that in the axis the distribution of potential is relatively the same as between plates extending to infinity on either side. This case, therefore, was considered, and the design based upon it. My present position, however, is that the design was justified by experiment.

Effect of Want of Centering of the Sphere or Ellipsoid within the Glass Tube. — The effect of a want of centering here is to aggravate the normal instability, and the effect on the observations was to immensely increase the uncertainty of the zero position owing to the increased electrification of the suspended dielectric. Care was always taken, therefore, to center the ellipsoid with respect to the tube as perfectly as possible, and this could be done by leveling in perpendicular planes with the requisite accuracy.

I may say that had I known the extreme variability of the quantity I set out to measure from the commencement, most of this work would have been unnecessary; but, at first, whenever I got two pieces of ebonite or sulphur to exhibit different effects, I assumed that the apparatus or mode of observing was at fault, and it was only after satisfying myself that this was not the case that I adopted the hypothesis of variability in sample, which afterwards turned out to be the most strongly marked feature of the phenomenon.

Influence of the Guard Tube. — The influence of the cylindrical guard tube in an originally uniform field can be calculated by means of Maxwell, Vol. II., § 432, together with the consideration that in a uniform field the strength of field within such a tube is the same as that within a spherical shell inscribed in the cylinder. Taking the origin of coördinates at the center of the shell, Fig. 2, and supposing that the direction of the field is in the plane xz and parallel to the x axis, it is clear that the influence of the sphere and shell are identical along that axis. But Maxwell shows that the field inside the sphere is uniform, and if the extension in the z direction is infinite, the field inside a cylinder must be uniform in virtue of symmetry: hence the field at any point inside the cylinder is the same as that inside the shell. This, of course, follows at once if we put the z axis of the ellipsoid infinite in Maxwell's investigation.

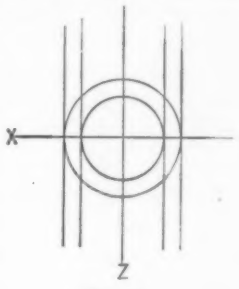


Fig. 2.

Making use of Maxwell's equation, § 433, we find that the force inside the cylindrical tube of radii of curvature a_1 and a_2 for the two surfaces is equal to the force outside multiplied by

$$\frac{9(1+4\pi K)}{9(1+4\pi K)+2(4\pi K)^2\left[1-\left(\frac{a_1}{a_2}\right)^3\right]}$$

where $K=\frac{\mu-1}{4\pi}$, where μ is the specific inductive capacity.

The above expression is, therefore, equivalent to

$$\frac{9\mu}{9\mu + 2(\mu - 1)^2\left(1 - \frac{a_1}{a_2}\right)^3}.$$

Now with the glass tube employed I had no means of determining μ, for it so happened that I had only about a foot in length of tubing of a suitable diameter and thickness of wall. This tubing was of flint glass, made by Powell, and had a density of 3.22. The specific inductive capacity of a flint glass of density 3.21 was examined by Hopkinson, or rather he examined the specific inductive capacity of Chance's light flint of density 3.21, and I have assumed that this would be practically identical with the glass I used. The value for μ found by Hopkinson was 6.85. Using the value 6.8, the value of the above expression becomes, in the case quoted, 0.786, which is hereafter referred to as the "screening factor." Now my object was primarily to investigate the laws of dielectric loss under similar circumstances in different dielectrics, and as the exact value of the screening factor only affects the very variable *absolute* value of the losses, it is not of any great importance. A little consideration, however, will show that the screening by the guard tube is a matter requiring close attention. In the first place, the field outside the glass tube is not rigorously uniform, consequently the method of calculating the screening does not rigorously apply. In the second place, the whole process is doubtless vitiated to some extent by the fact that the polarization of the material of the guard tube is not exactly in phase with the electric intensity any more than it is in the suspended dielectrics. The result of this action is to give, so far as effects of the second order are concerned, an exceedingly complex field inside the guard tube, the nature of which it is not easy to forecast exactly. However, the chief effect will be to cause the rotating field within the guard tube to lag slightly behind the field corresponding to air alone, and, at the same time, to slightly reduce the screening effect. Now the experiments to be described, as well as those of Arno already quoted, show that the magnitude of the dielectric losses depends on the speed of field rotation; so that, strictly speaking, a special screening factor should be employed for each separate speed of rotation. It appeared that nothing but an extensive series of experiments could afford the required evidence as to the complex effects of screening. Accordingly, the months of November and December, 1895, were taken up in investigating this matter in the following way. Three guard tubes were used: the one already mentioned, one of thin ebonite, and one of mica — bare, varnished, and paraffined. Complete series of observations were made on ellipsoids of glass, ebonite, and sulphur at potential differences of the plates

from 250 to 6000 volts, and even to 12,000 in some cases. The speed of rotation varied from 5 to 6 per second, up to 28, the greatest frequency attained. The results of observation were first plotted simply as curves of angular deflection and potential difference, and afterwards by plotting the logarithms of these variables. The reproduction of the laboriously acquired results, however, would serve no useful purpose, and I will, therefore, merely state that the logarithmic curves were in all cases straight lines, and that their regularity was such that on submitting them to my friend, Mr. Knibbs, an expert in the reduction of observations, he was able to assert that, so far as the numbers went, the value of the index n in the equation $D = \alpha V^n$ (D, angular deflection of dielectric; V, potential difference of plates) could be relied upon to about unity in the third significant figure. The curves referring to the same dielectric in the three guard tubes were, as nearly as could be seen, exactly parallel; and though the intervals were not quite the same at high as at low speeds, the differences were not worthy of attention from the point of view of the present inquiry. It also appeared, as was to be expected, that the hygrometric state of the air had a very large influence on the result; and, as this could not be very well controlled, one result of the investigation was to show that it was useless to make measurements except in dry weather, and that it was very necessary to have plenty of drying material inside the guard tube.

The calculated screening factors were:

Glass tube	0.786
Ebonite tube	0.966
Mica tube	1.000

With regard to the deflections in glass and ebonite: In a certain experiment which may be selected as having involved high voltages, the deflection of a sulphur ellipsoid in the glass tube was compared with its deflection in the ebonite tube. The calculated ratio of deflections was 1.50 to 1, and the observed ratio was 1.54 to 1.

In the case of the mica tube the results were less satisfactory, but as this was traced to the poor insulating power of the only mica of suitable size I could procure, nothing need be said about them.

The glass tube was finally selected as the more suitable on account of its transparency, and the whole of the subsequent observations were made in it. Up to this point I had enjoyed the advantage of the assistance of my friend, Mr. Pepper, to whom I hereby desire to express my indebtedness.

Effects of Moisture. — The invariable effect of a damp day was to decrease the deflections, owing to the extra screening of the guard tube. By drying the tube externally by hot dry air the deflections could be increased

for a short time. The action of the rotating cell is to produce an enormous down draught all round the guard tube, so that unless the air is dry, *i.e.* contains not more than about 60 per cent of the maximum moisture, no reliable absolute results can be obtained. The only remedy is to abstain from working in damp weather; and after this fact was discovered I made no measurements when blowing in hot air was found to have an appreciable effect. It was afterwards found, however, that different samples of the same substance, tested under exactly similar conditions, gave results differing much more from one another than any difference produced by a rise of humidity to, say, 75 per cent, and consequently a large number of observations which I at first threw aside as likely to be affected by moisture have a certain value.

Effect of Want of Centering of the Suspended Dielectric. — Owing to the mode of preparation adopted, this never occurred to an appreciable extent.

Effect of Want of Perfection of Form in the Suspended Dielectric. — When the period of rotation of the field is small compared with the period of vibration of the ellipsoid, want of precision of form has no direct effect on the rotation produced. This was roughly checked by comparing the behavior of two cylinders of ebonite, one of which was perfect in form and the other intentionally mutilated by filing. Of course, there is an indirect effect in so far as the actual electric intensity inside the ellipsoid depends upon its shape.

Effect of Want of Perfect Drying of the Dielectric. — I had hoped that the absence of points of sudden variation of electric intensity over the surface of the dielectric would have removed the necessity for drying. This hope, however, proved to be vain. The effect of the slightest moisture was generally to increase the deflections for a given potential difference, so that in most of the experiments the material was left in presence of phosphorus pentoxide for at least three days, and the readings were not passed till they became invariable. On the other hand, the effects of paraffin and shellac varnish on the deflections of glass, sulphur, and ebonite ellipsoids were studied, with the result that differences of the order of a few per cent only were ascertained; that is, of course, when the drying was thorough. Great pains were taken over this part of the work, as the whole object of the investigation was to separate effects due to creeping from the other phenomena. The sort of agreement observed may be illustrated by an example of the behavior of an ebonite sphere of 1 cm. diameter at a frequency of 16.

The reason for the fall observed in the observations 2 and 3 is the slight but inevitable damping of the inside of the guard tube by the admission of damp air (the humidity was 72 per cent). In the second and third obser-

vations the ebonite was dried by flame gases before it was put back in the tube, and this was done while it was warm, *i.e.* at about 40° C. in both cases. Many cases both of better and of worse agreement could be cited, but this is a fairly typical one.

TESTS OF AN EBONITE SPHERE UNDER VARIOUS CONDITIONS OF ITS SURFACE.

State of surface.	How dried.	P. D. of cell.	Rotation observed.
1. Paraffin.	Exposed to the action of P_2O_5 for 48 hours.	3500	253′
2. Paraffin washed away by benzine.	Dried by hot air, and placed while hot in the guard tube.	3500	207′
3. Reparaffined.	Dried by hot air, and placed while hot in the guard tube.	3500	204′

Methods of Observing.

The method of making observations was as follows: The conditions as to dryness having been found to be satisfactory, the cell was started and the deflection of the tachometer noted. This was reduced to absolute value by means of a curve, and was, when necessary, further checked by an observation at the myograph.

The zero of the dynamometer was noted, the earth connection broken, and the cell raised to a high potential. A shift of zero indicated insufficient screening of the mirror and vane, and when this occurred the apparatus required attention. If no shift of zero appeared the earth connection was restored and observations taken at every 5 divisions of the voltmeter, *i.e.* at intervals of 500 volts. It was the business of the assistant at the regulator to keep this constant. If the needle moved off the mark to the extent of half a division on either side, he gave a signal and the observer at the dynamometer (myself in all cases) suspended operations. The practice was to take ten elongations of the light spot at each potential difference — five in each direction — and some time was allowed both for reasons already stated, and to eliminate any want of attention to the adjustment of the potential difference. As a rule, the observations were carried from 0 to 60,000 volts, after which the cell plates were connected first by a high and then by a low resistance, and the dynamometer zero was reobserved. In many cases no shift whatever was noted, and no series of observations for

standard purposes was passed until this condition became satisfactorily satisfied — it was, with some exceptions, chiefly a question of drying. The cell was then stopped, its motion reversed, and the same series of observations gone through. The deflections were then reduced to "single" deflections, *i.e.* the actual rotation of the spheroid, and expressed in minutes of arc by reference to a curve; and the logarithms of these rotations and of the potential differences were plotted out on squared paper.

TREATMENT AND REDUCTION OF OBSERVATIONS.

Let θ be the rotation in minutes of the ellipsoid under the influence of a potential difference V (measured in units of a certain number of volts, generally 100). Then by plotting logarithms it was found that, within the limits of errors of observation, the relation could be represented by a formula of the kind $\theta = \alpha V^n$, where α and n are constants.

n was determined from the plot, and then a pair of simultaneous values selected from the curve gave α.

It is required to reduce the relation thus obtained to electrostatic measure, and express the waste of energy per cycle in ergs as a function of the electric intensity inside the dielectric. The perpendicular distance of the plates of the cell was 3.8 cm., and the screening factor for the glass tube was 0.7865.

Let S be the numerical factor required to reduce the external electric intensity to the intensity inside the ellipsoid.

Let F be the internal electric intensity.

Then, with the above data and the value of the ratio of the electrostatic to the electromagnetic units,

$$F = 0.069\, SV = qV. \qquad (2)$$

The value of S is calculated from the shape of the material used and from its specific inductive capacity — exactly as in the magnetic case — for which Ewing's *Magnetism in Iron and Other Metals* may be consulted.

There is no need to reproduce the argument here, but it may be mentioned that the *correction* is important.

For instance, for a certain spheroid of ebonite with diameters 2.646 and 0.984 (suspension being about the longer diameter, the value of S was 0.396, and for a sulphur ellipsoid of similar form $S = 0.43$.

The energy lost per revolution is evidently proportional to the angular deflection of the ellipsoid. Let W be the energy loss per unit volume per revolution, *i.e.* per cycle of electrification. We may write

$$W = k\theta, \qquad (3)$$

where k is a constant.

Let ω be the volume of the ellipsoid.

Let the thread be vibrated by a body of moment of inertia m, and have a period of vibration t, and let p be the value of one minute of arc in circular measure.

Then
$$k = \frac{4\pi^2 mp}{\omega t^2}. \qquad (4)$$

Then, from (1), (2), (3),
$$W = ka\left(\frac{F}{q}\right)^n = CF^n.$$

Let K be the specific inductive capacity of the dielectric employed. Then the energy per unit volume due to the electrification is
$$\frac{KF^2}{8\pi}.$$

There is no meaning to be attached to the phrase "percentage of energy lost per cycle," because n is not in general equal to 2. Since, however, the experiments to be described give values for C and n, the percentage loss can be calculated for any assigned value of F (of course within the limits covered by the experiments).

One illustration of the methods of calculation will suffice. I select at random the following:

Sulphur Ellipsoid K. — The sulphur was composed of what I have already described (Royal Soc. paper unpublished) as "aged monoclinic sulphur" combined with 0.708 per cent of insoluble sulphur. I say "combined" advisedly as opposed to "mixed."

Major diameter of spheroid, 2.55; minor diameter, 0.97; mass, 2.5772 g. Specific inductive capacity 3.2 (*loc. cit.*); volume, 1.29 cc.; frequency of field rotation, 18 per second.

The observations gave $n = 1.857$, and the relation
$$\theta \text{ (in minutes)} = 0.143\ V^{1.857},$$
V being in 100 volt units.

The value of S in this case is 0.43. The moment of inertia of a cylindrical vibrator was 0.079; and the period of vibration 30″.

Using these data we find
$$W = 0.00048\ F^{1.857}.$$

In the case where $F = 1$, this gives a percentage loss of about 0.0038 per cent per cycle, at 16 cycles per second.

As it happens, the observations leading to this relation were about as irregular as any ever admitted, and I will, therefore, tabulate them. It is no use to draw the logarithmic curve, for when reduced to the size of these pages, all the observations would probably appear to fall upon the

line. In many cases, using other substances, even when plotted on $\frac{1}{10}$-inch paper, on a scale of 10 inches to unity in the logarithms, the errors were of the same order as the error of ruling of the paper, and could not be detected without close scrutiny.

READINGS OF VOLTAGE AND DEFLECTION.

Sulphur Ellipsoid K. May 12, 1896.

Voltmeter readings at 100 volts per division.	Reading of dynamometer. Rotation counter-clockwise. Number of rotations 18 per second.	Reading of dynamometer. Rotation clockwise. Number of rotations 18 per second.	Double deflection in scale divisions; i.e. centimeters at 96 cm. from mirror.	Rotation of ellipsoid in minutes of arc.	Log: minutes.	Log: (100) volts.
10	18.13	17.09	1.04	10.0	1.000	1.000
15	18.37	16.76	1.61	15.0	1.176	1.176
20	20.07	16.16	3.91	35.0	1.544	1.301
25	21.41	15.37	6.04	54.0	1.732	1.398
30	23.17	14.35	8.82	78.5	1.895	1.477
35	25.21	12.80	12.41	111.5	2.047	1.544
40	27.21	11.61	15.60	139.0	2.143	1.602
45	29.00	10.08	18.92	168.0	2.225	1.653
50	31.37	8.70	22.67	202.0	2.305	1.699
55	33.76	7.40	26.36	234.5	2.370	1.740
60	37.40	5.64	31.76	282.0	2.450	1.778

The observations at 4500 volts were selected from the curve for calculation.

EBONITE.

A large number of samples of this substance were examined, and it was found by their aid:

(1) That the index n of the curve in the equation

$$W = CF^n$$

is independent of the nature of the guard tube, or the state of dryness of its surface, but that, on the other hand, it depends almost entirely on the dryness of the ebonite surface.

(2) When the speed is varied the index remains constant, though the value of C varies a little with a guard tube which does not insulate perfectly. Though I never entirely succeeded in eliminating this effect, it became, under suitable circumstances, so small that I think there is no doubt that with the samples examined there is no change of n as the

speed of rotation increases from about 10 to 30 per second. Even in those cases in which C appeared to vary there was no variation of n.

(3) The constants are independent of the nature of the surface, *i.e.* it does not matter whether the ebonite is polished dry; is smeared with hard paraffin or vaseline; or is rough, or varnished with shellac varnish, so long as the surface is in good insulating condition.

(4) No two samples of ebonite have given the same result, though the same sample may be dismounted, remounted, and examined under circumstances as varied as possible without giving variable results.

The following data may be regarded as typical:

Ebonite spheroid	"Long A."
Longer diameter	2.646 cm.
Shorter diameter	0.984 cm.

The spheroid was suspended about its longer axis.

Weight of spheroid	1.610 g.
Volume of spheroid	1.382 c.c.
Specific inductive capacity, assumed	3.5.
Internal screening factor	0.396.

After being dried by phosphorus pentoxide for 14 days, and on a day when the air contained 45 per cent of the possible moisture, I found

$$W = 0.029\, F^{1.765}.$$

Percentage loss, 21.2 when $F = 1$.

Percentage loss per cycle, $21.2\, F^{-0.235}$.

This was at a field rotation of 15 revolutions per second. On varying the speed and keeping the potential difference (5000 volts) constant, the following deflections were obtained:

Field revolutions per second.	Double deflections in centimeters.
11	31.97
15	31.82
30	32.21

The very slight increase at the higher speed is, possibly, to be attributed to the effect of a slight residual conductivity of the surface of the guard tube.

UNIVERSITY OF SYDNEY, NEW SOUTH WALES,
November 23, 1896.

[*The conclusion will appear in the following number of the* PHYSICAL REVIEW. *The article is divided arbitrarily at this point.*]

MINOR CONTRIBUTIONS.

THE APPLICATION OF THE INTERFEROMETER TO THE MEASUREMENT OF SMALL ANGULAR DEFLECTIONS OF A SUSPENDED SYSTEM.

BY F. L. O. WADSWORTH.

IN an interesting paper in the last number of the PHYSICAL REVIEW, "On the development of the Idiostatic Electrometer," Professor Barus has described an application of the Michelson "interferometer" to the measurement of the small *linear* movements of the electrometer disk, and has suggested the use of the same instrument in the measurement of the *angular deflections* of an ordinary suspended system. If such an application is practicable, it is at once evident what a field of usefulness is open to the interferometer in the measurement of minute deflections, which our present means of reading do not enable us to measure with any degree of accuracy. It may therefore not be uninteresting in this connection to state that this application has already been made, and its practicability under proper experimental conditions demonstrated. The difficulties encountered, however, particularly by those who should attempt to use the instrument for the first time, are even greater than those indicated by Professor Barus. Indeed I have found that many find difficulty at first in finding and adjusting the fringes at all, even when working with the most stable form of instrument with all the mirrors fixed. These difficulties are rendered greater than they need be by the fact that, although the theory of the instrument has been fully discussed and explained by both Professor Michelson and Professor Morley,[1] and the general types and special forms for both linear, angular, and spectroscopic measurements described by Michelson in a number of papers,[2] there has not been pub-

[1] See paper, Interference Phenomena in a new form of Refractometer, A. A. Michelson, Phil. Mag., Vol. 46, p. 395, 1882; Théorie de L'Appareil D'Interférence, A. A. Michelson, Trav. et Mem. du Bureau internat. des Poids et Mesures, Annexe II., Tome XI., p. 115; On the Measurement of the Expansion of Metals by the Interferential Method, Part I., Optical Principles on which the Method depends, E. W. Morley, PHYSICAL REVIEW, Vol. IV., p. 3. Also joint papers by Michelson and Morley, On the Feasibility of making the Light Wave the Ultimate Standard of Length, Amer. Jour. Science, Vol. 38, Sept., 1889, and Journal of Association of Eng. Soc., May, 1888.

[2] Measurements by Light Waves, Amer. Jour. Science, Vol. 39, p. 115; Application of Interference Measurements to Astronomical Measurements, Phil. Mag., Vol. 30,

lished, so far as I am aware, any description of those details of construction and manipulation which seem comparatively unimportant to those rendered thoroughly familiar with the instrument by long use, but which nevertheless are likely to occasion difficulties to those taking up the construction or the use of one of these instruments for the first time, particularly when, as in this case, they are not commercially obtainable in standard forms. For this reason I take this occasion to go over the general construction and details of adjustment common to all forms of interferometer before describing the special forms of instrument with which the experiments on the measurement of angular deflections of a suspended system were made.

In his paper "Measurements by Light Waves" in the *American Journal*, Michelson divides interferometers into five general groups, the distinction between which in some cases does not seem very clear. It has always seemed to the writer that a division into two general classes, according to the character of the source of light with which the instrument is to be used, would be better and more comprehensive. On this basis the first class would include all those forms of interference apparatus in which the source is necessarily of very small angular magnitude; types of these being Fresnel's and Loyd's mirror systems, Billet's bi-lenses, and Michelson's "telescope refractometers."

The second, and by far the more important class, comprises those instruments which may be used with a source of any angular magnitude, *i.e.* an extended luminous surface. In this class, of which the earliest example was Jamin's refractometer,[1] there is necessarily a plane of separation, generally an optical surface of some transparent material, at which the incident light is divided into two beams, one reflected, the other transmitted; which pass away from the surface in different directions, and are afterwards brought together again by any desired combination of reflecting surfaces or lenses. If the transmitted ray is allowed to emerge from a second surface parallel to the first (surface of separation), it is evident that we may, by simply changing the inclination of the latter to the incident light, obtain any angular divergence of the two separated beams from 0° to 180°. Figs. 1, 2, 3, and 4 show four forms in which the separation β of the two rays is respectively 45°, 90°, 135°, and 180°.

In these figures the single heavy arrow indicates the beam of incident light, the two thin arrows the reunited rays which produce interference. The shaded plates are mirrors (silvered on front surface), the unshaded

p. 1; Application of Interference Measurements to Spectroscopic Measurements, Part II., Phil. Mag., Vol. 34, p. 280; and Smithsonian Contributions to Knowledge, Application of Light Waves to Metrology, Nature, Nov. 16, 1893.

[1] Ann. de Chem. et de Phys., 3d Ser., Vol. 52, p. 163, 1858.

plates are plane parallel glasses, the heavy shaded side of the one marked A being the side at which the separation of the two rays takes place. In Figs. 1 and 2, the two rays are reunited at this same surface; in Fig. 3, at a second surface, A'.[1] In the former case a compensator C is required, in the latter none is needed.[2]

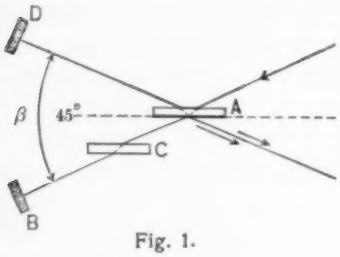


Fig. 1.

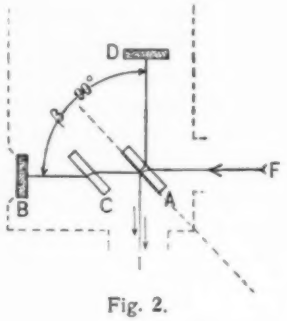


Fig. 2.

It is evident that an infinite number of individual forms may be produced by changing the angle β and the subsequent arrangement of the mirrors which bring about reunion. None of these, however, differs in any essential particular either as regards the method of mounting and adjusting the mirrors, or the character of the interference phenomena produced. As

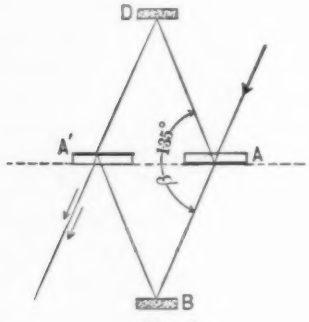


Fig. 3.

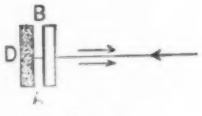


Fig. 4.

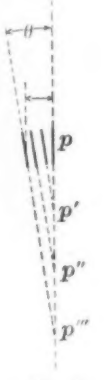


Fig. 5.

[1] In this case the function of the surfaces A and A' may of course be interchanged; *i.e.* A' may be the surface of separation, A that of reunion.

[2] The exact correspondence between this case and any of the others shown may be easily seen by projecting the path of the reflected ray on the path of the transmitted one by cutting the paper along the dotted lines (the prolongation of the intersection of the separating surface A with the plane in which the angle β is measured) and folding the parts to the right over upon the parts to the left. This process also makes clear the part played by the compensator C in making the two paths optically the same.

will be most readily seen from Figs. 4 and 5, this is simply that produced by reflection when the light falls upon a thin air film whose thickness at the center is a, b, and whose sides are inclined to each other at an angle θ. When the angle θ is 0, as in Fig. 4, the interference fringes are circles whose diameters are determined by the distance a, b (the thickness of the film); when θ is not zero, as in Fig. 5, the fringes are hyperbolas or parabolas which are symmetrically arranged on each side of a single straight fringe lying at the line of intersection p of the two sides of the film.[1]

In all the instruments which have so far been used for either angular or linear measurements, the general arrangement shown in Fig. 2, for which $\beta = 90°$, has been adopted either in the exact form there shown or with some simple modification for securing (as in the case of the Paris interferometer) greater economy of construction and greater convenience of manipulation; or (as in the case of the form adopted for the angular measurements presently to be described) a more symmetrical arrangement of parts about the axis of rotation. This form may therefore be taken as the general type of construction for all instruments of this class.

In the descriptions of the details of setting up and adjusting the interferometer which follow, I shall assume that the reader has made himself familiar with the general theory and method of operation of the instrument from the papers already referred to. For convenience of description I will indicate the four principal optical parts by the following letters (see Fig. 2):

A indicates the separating surface.
C indicates the compensating glass.
B indicates the fixed mirror of the interferometer. Sometimes called the reference plane.[2]
D indicates the movable mirror of the interferometer.

The difference in path between the two rays is in every case twice the thickness of the equivalent air film ab. This will be indicated by the symbol Δ.

In regard to the general design of the instrument the following points must be considered:

1. The bed of the whole instrument should be very heavy and rigid in order to afford an unusually steady and stiff support for the whole mirror system. It should be H or U shaped in section, of a width of at least twice the aperture of the mirrors B and D, of a height at least one and one

[1] For fuller discussion of these phenomena see papers already referred to, particularly that of Michelson's in the Phil. Mag. for 1882, and that of Morley's in this journal: Vol. IV. p. 3.

[2] In Fig. 4, A performs also the function of B.

half times the width, and of a length depending on the range of movement allowed for the movable mirror. The extreme difference of path Δ at which the interference fringes have yet been observed is about 45 cm. (with the light from the green mercury line), and since, as just stated, the difference of path introduced by the movement of D is twice the movement itself, it is not generally necessary to provide for a greater range of motion than 20 cm. The carriage upon which D is carried should be long (three to five times its width), in order to minimize the effect of error in the ways. The best form for the latter is that shown in Fig. 6,[1] which is a cross section through the bed, movable carriage, and mirror. These ways should be as true and smooth as possible, a good way of securing the desired result being to grind them by the method described in a recent paper in the *Journal of the Franklin Institute*.[2]

The screw used to move the carriage does not usually need to be an especially accurate one, but it should work very smoothly in the nut.

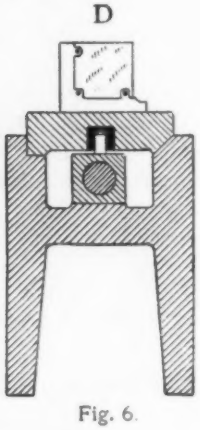


Fig. 6.

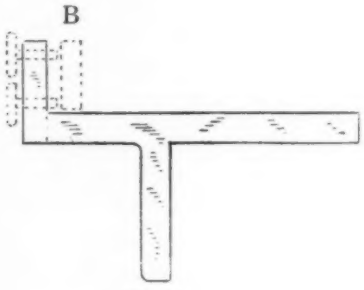


Fig. 7.

The nut may for the same reason be a simple square solid nut, as shown, with its lower side sliding against the surface of the cross bar of the bed, and with a pin projecting from its upper surface and entering a slot in the carriage. This slot should be considerably wider than the diameter of the pin, so that when the screw is turned backward a half turn no part of the nut will touch the carriage. At the forward end the screw should be provided with a graduated head, and also some simple form of index for indicating the whole number of turns. It is con-

[1] This is the form adopted for the Paris interferometer. For a discussion of its advantages see paper On the Manufacture of Very Accurate Straight Edges, F. L. O. Wadsworth, Jour. Franklin Inst., July, 1894, and American Machinist, Aug. 23, 1894.

[2] *Ibid.*

venient to have the screw itself of a comparatively coarse pitch (not finer than 1 mm., and better 2 mm.), and have at the front end a slow motion device, such as a tangent screw or worm gear that may be readily thrown in when desired to give a very slow motion to the carriage.

The head plate at the front of the bed which carries the glasses *A*, *B*, and *C* (see Fig. 7) should be thick and heavy (not less than one and one half to two times the thickness of the glass plates themselves), and, to prevent springing, should be screwed to the bed on the side only.

2. *Mountings of the Mirrors.*—The proper mounting of the mirrors is of the utmost importance, as upon this more than upon anything else depends the successful and convenient manipulation of the instrument. They must be held firmly and at the same time in such a way as to introduce no strain, and so as to allow each one to be independently adjusted in both a vertical and horizontal plane. The most convenient and efficient way which I have

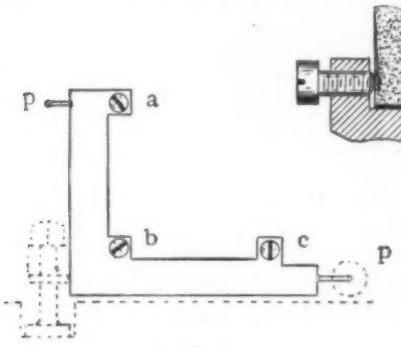


Fig. 8.

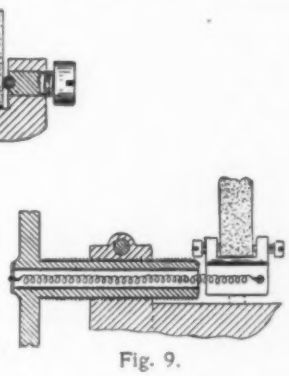


Fig. 9.

found of fulfilling these conditions is to support each mirror (if rectangular, as is generally the case) on an L-shaped piece at whose vertex and ends are three inwardly projecting forks (see Figs. 8 and 9), the opening in which is slightly greater than the thickness of the mirror. The whole piece is milled out of the solid metal, and its thickness is about one and one half times the thickness of the mirror, so as to be very stiff and rigid in the direction at right angles to the plane of the mirror. Through each arm of the forks are threaded fine screws, *a*, *b*, *c*, between the points of which the glass is lightly clamped.[1] On one side the glass rests directly against the points of the screws; on the other the points are hollow, as shown in the figure, and little rounded plugs of cork are inserted to allow for any expansion or contraction, and to allow also of a slight adjustment of the glass without moving the screws on both sides.

[1] One great advantage of this form of mounting is that mirrors of different sizes may be held in the same mounting.

In the case of glasses *A* and *D* the supporting piece is screwed down to the head plate and carriage respectively, by means of screws through the lower arm of the L; but in the case of *C* and *B* it is mounted so as to permit of a more delicate motion of adjustment than can be obtained by means of the screws *a* and *c* alone. This may be and has been done in a number of ways in different instruments, but the plan I have found most reliable and convenient is one similar to that used in the Paris interferometer for the back support of the "intermediate standards," and for the support of the compensating mirror. This is to mount the L piece on a short, stiff steel spring, a convenient form of which is a steel column, one end of which is screwed into the bed plate and the other into a lug on the side of the L near the vertex (see Fig. 8, dotted lines). The required small horizontal adjustment is effected by slightly *twisting* this column about its axis, best by the varying pressure of a very light spring acting on the end of the horizontal arm of the L; the vertical adjustment by *bending* the column by a similar arrangement acting on the end of the vertical arm.

The most recent arrangement which I have used for this purpose, and which has been found more compact and satisfactory than any heretofore tried, is that shown in Fig. 9. It consists of a light spiral spring, one end attached to a pin in the end of the arm of the L, the other to the outer end of a hollow screw through which the spring passes freely. The thread of the screw should not be finer than 1 mm. pitch,[1] and should work very smoothly in the nut, which should be at least two diameters in length. The strength of the spring should be so adjusted that one revolution of the screw shifts a fringe by not more than twice its own width. The screws should have a run of about 2 cm., corresponding to a total movement of about 40 fringes, or for a mirror 2 cm. wide, to a linear movement of about 0.01 mm. It will be at once seen by those familiar with the inherent defects of even the very best micrometer screws, how very difficult it would be to move the mirror by an amount corresponding to a tenth of a wave (*i.e.* 0.000025 mm.), as is necessary in the final adjustment, by any screw bearing directly upon the mirror or any arm of practicable length attached thereto. Not only do the errors of the screw far exceed this amount, but what is more, the unavoidable longitudinal pressure exerted by the fingers in turning the head, moves the screw in the nut and springs the support of the latter, no matter how rigid we may attempt to make it, by an amount far greater than the desired movement. With this spring arrangement the effect of these sources of irregular motion is reduced in the same proportion that the movement of the screw for a given effect is increased, *i.e.* about 1000 times. To reduce the effect in the same proportion by

[1] Heretofore it has been customary to use screws of much finer pitch bearing against much stiffer flat steel springs.

simply lengthening the arm of the L would require that this be made 20 m. long.

In the case of the compensating glass there is no necessity for a delicate adjustment in the vertical plane, and only one spring movement, *i.e.* that acting on the end of the horizontal arm of the support, is needed. In this case we need a movement corresponding to only two or three fringes, but since this movement is used to measure fractions of a wave length, the inequalities in the motion must not exceed a quantity corresponding to $\frac{1}{100}$ wave. The pitch of the screw should therefore be much coarser than before (a double or triple thread or spiral of about 5 mm. pitch is not too coarse), and the spring still finer, so that one complete turn corresponds to a shift of one fringe. This screw should have a large head, graduated to hundredths of a turn (so that one division corresponds approximately to $\frac{1}{100}$ wave length).

Setting up and Adjusting the Instrument.

a. Silvering of Mirrors. — Mirrors *B* and *D* are full silvered on their front surface; *A* is "half silvered" on the surface nearest the compensating glass. Of the different processes of full silvering, the writer prefers above all others that of Brashear, the details concerning which were given in the *Astrophysical Journal* for March, 1895.[1] For "half silvering," *i.e.* depositing a semitransparent coat of such density as to reflect about half the incident light, I have, however, obtained the best results with the ordinary rochelle salts solution prepared as directed by Boettger.[2] For "half" silvering the solution should be comparatively new, and the glass cleaned with the greatest care in order to obtain a uniform film. The plate should be supported in the liquid by one of the glass "forks" described in the article above referred to, so that it may readily be lifted out of the solution from time to time and the density of the deposit examined. When of about the right thickness it has a peculiar blue color by transmitted light. A few trials will be necessary at first to enable the operator to determine by this means just when the right thickness of film has been obtained. It should then be immediately lifted from the silvering solution, plunged into a large vessel of distilled water and thoroughly rinsed, and then turned on edge on a sheet of blotting paper to dry. Great care must be taken not to touch the silvered surface with the fingers either before or after drying, as it is so thin that it is very readily rubbed off.

b. Putting Mirrors in Place. — First put *D* and *B* into their supports,

[1] "Silvering Solutions and Silvering"; F. L. O. Wadsworth, Astrophysical Journal, Vol. I., p. 252.

[2] Kohlrausch, Physical Measurements, p. 146.

and by means of a small steel square adjust them so that their surfaces are at right angles to the side and end of the bed, and the faces of the carriage and head plate, respectively. Then if the mechanical work on the machine has been properly done these two surfaces ought to be very nearly at right angles to each other. Next put in *A* and *C* and adjust *A* until the image of *D* seen by reflection is exactly superposed on *B*; then finally adjust *C* until it is parallel to *A*, either mechanically by the use of a pair of outside calipers (outside calipers are used instead of inside ones to avoid injuring the silver film which is on the side nearest the compensator), or optically by making the two images of a bright distant object seen by reflection from the surfaces of *A* and *C* coincide. It is, perhaps, necessary here to call attention to the fact that the thickness of the two plates *A* and *C* should be the same within a few thousands of a millimeter, and that both ought to be as nearly as possible plane parallel, *i.e.* both surfaces should be optically flat, and as nearly parallel to each other as possible. The best way is to have them both cut from one large piece.[1] If it is necessary, as sometimes happens on account of the loss or breaking of one of the "pair," to use two pieces of slightly different thickness, we may up to a certain point still obtain practically perfect compensation by inclining *C* slightly to *A*, so as to make the length of the path traversed by the rays in the glass equal in the two cases. If *C* is thicker, it is of course turned so as to make the angle of incidence smaller than on *A*; if thinner, so as to make the angle of incidence larger. It is further to be mentioned that these adjustments are only necessary in order to obtain the fringes in white light; with monochromatic light we might dispense with *C* altogether, as is sometimes done.

c. To Find the Fringes. — First measure, by means of a thread or scale, the distances from any point on the edge of the silvered surface of *A* to the surfaces of *B* and *D*, and move the latter forward or backward until these distances are the same to within about 1–4 mm. Then cover the whole instrument with a wooden or pasteboard box (as indicated by dotted lines in Fig. 2), having two holes about 3 cm. square cut in front of the mirror *A*, as shown. It should also have an opening in front through which projects the handle and the slow-motion device for turning the screw, and another opening in the side opposite the mirror *B*, through which the screws which adjust the latter can be reached. This box is not essential if the room is very quiet and free from air currents, but it is always desirable to use it when possible, as it will frequently greatly facilitate the finding of the fringes. Next place a sodium flame at *F*, about 30 or 40 cm. from *A*,

[1] Mr. Brashear has attained such skill in the manufacture of these plane parallel pieces that he can furnish at merely nominal cost a "pair" that will always give satisfaction.

and between the two an ordinary condensing lens which will render the rays falling on *A* approximately parallel. Place the eye at *I*, in front of *A*, looking in the direction of the double arrow toward *D*, and move the flame and condenser until the whole field of the mirrors is uniformly filled with light. Place a needle point just in front of the flame near the focus of the condenser, and with the fine motion screws of *B* at the middle of their run, adjust this mirror with the screws *a* and *c*, until the two images of the point seen by reflection in *A* (one formed by reflection from *D*, the other by reflection from *B*) are in exact coincidence. (Besides the pair of images formed by reflection from the silvered surface of *A*, there will be another pair formed by reflection from the front unsilvered surface. If the glass is exactly plane parallel this second pair will also be in coincidence when the first pair is, but not otherwise. Care, therefore, should always be taken to select the right pair of images. They may be easily distinguished by their greater brilliancy.) This adjustment to coincidence needs to be performed with great care, for unless it is very exact the fringes will not appear. It is, therefore, best to use at first a telescope at *I*, by means of which the images of the point will be considerably magnified, and the coincidence more easily determined. It is also a help to change the position of the needle, holding it with its axis horizontal when adjusting in a vertical plane, by means of the screw *a* (Figs. 8 and 9), and vertical when adjusting in the other plane by means of screw *c*. When coincidence is judged to be exact, remove the telescope (if one has been used) and needle point, and place the eye about 25 or 30 cm. from the mirror *A*. If the fringes do not at once appear (generally they will appear as fine, more or less curved lines), first move the eye back and forth (there being a position of maximum visibility for each particular value of Δ and θ). If this does not bring them into view, move *D* slightly (about 0.2 mm. each time) by means of the screw, first in one direction and then in the other, and examine again after each movement by moving the eye backward and forward as before. After each movement of *D*, reëxamine the images of the needle point to see that they are still in coincidence. The fringes will be certain to appear in one of these positions if the preceding directions have been carefully followed.

d. Adjusting the Fringes. — After the fringes have once been seen, it will be found an easy matter, after a little practice, to adjust them to any desired width or position by means of the fine-motion screws on *B*. A little manipulation of these screws and a study of the effect produced is worth more than any number of directions. A word, however, may be said regarding the adjustment for circular fringes, which appear when each of the mirrors is exactly perpendicular to the incident beam of light falling on it; and the finding of the central straight fringe in white light. To

produce circular fringes, the mirror D should be moved back until there is a considerable difference of path between the two rays (1000 to 2000 wave lengths), and the screws moved so as to *increase* the curvature of the fringes. The center of the system will then soon appear, and may, by a further movement of the screws, be brought to the center of the field. Then move the eye to the right and left, and if the diameters of the circles change, move the screw acting on the horizontal arm of the mirror mounting B in such direction that this change grows less and less, and finally nearly disappears. Then move the eye up and down, and if a corresponding change takes place, adjust the other screw until this also disappears. When the fringes remain the same in all parts of the field, the adjustment is perfect.

To find the fringes in white light, the fringes are first made straight and preferably vertical. The screw is then turned so as to move the mirror D towards the zero position. It is easy to determine when this position has been nearly reached by the increasing straightness of the fringes, and also by the fact that there is very little tilting or change of curvature as the eye is moved up and down. The slow-motion screw is then thrown in, the sodium light replaced by a gas flame, and the carriage moved very slowly (at such speed that about 10 fringes cross the field per second) until the colored fringes appear. This position, as determined by the graduated head of the screw, should be noted for convenience in finding these fringes again when desired. Since only some thirty fringes are distinctly visible in white light, the position of the mirror must be right to within less than 0.01 mm. in order to make them appear at all.

e. Observing and Measuring with the Fringes. — The straight fringes obtained when the two mirrors are slightly inclined to each other are in general most conveniently observed by the eye alone, without the aid of the telescope. If a telescope is used it must be focused for the central white fringe on the surface D. For the fringes to the right or left of this, corresponding to a definite difference of path Δ between the two rays, the focus of the telescope changes. This fact, together with the continual change in the curvature, and consequent apparent shifting of the fringes with a change in the position of the eye, makes their use unsuitable in measuring any large differences of path. The circular fringes are free from all these difficulties, for being formed at infinity (see theory of instrument) they are always visible for all values of Δ in a telescope focused for parallel rays; and their position and appearance are likewise independent of the position of the eye. They are therefore always to be used when large differences of path are to be observed. For measuring small movements, however, the straight fringes are more convenient, it being only necessary to count the number which pass a given point in the field (most conven-

iently a line engraved on the surface of one of the mirrors B or D) when the movement takes place. When the displacement is so rapid that the counting is impracticable, the central fringe of the colored system seen in white light must be used to mark the initial and final position, this being the only fringe of the system that can be identified.

It may appear from what has preceded that the interferometer is an instrument of such extreme delicacy that its use in ordinary laboratory measurements is impracticable. Such, however, is not the case if the instrument is properly constructed. Its stability and steadiness is then perfectly surprising, considering the minute movements which it will detect. It is far less difficult to work with, for example, than a delicate galvanometer or electrometer, and in applying the interferometer to the measurement of the deflections of these instruments, the difficulties we meet with are not those due to the interferometer proper, but those due to the unsteadiness of the suspended system of the galvanometer or electrometer. It was for the purpose of determining whether these difficulties could be overcome that the experiments already alluded to were, at Professor Michelson's suggestion, undertaken, in 1891, at Clark University, by Dr. Austin, now of the University of Wisconsin, and the writer.[1]

The type of optical system adopted was that first described by Professor Michelson in his paper "Application of Interference Methods to Astronomical Measurements" (*Phil. Mag.*, July, 1890), and shown in Fig. 11

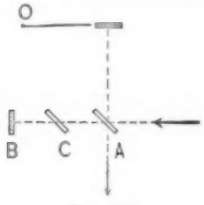


Fig. 10.

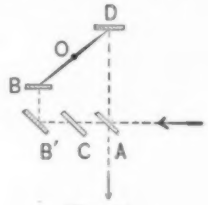


Fig. 11.

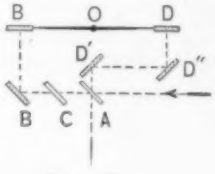


Fig. 12.

of that paper. Figures 10, 11, and 12 show how this is derived from the type of instrument for linear measurement. It is evident that this latter may be used as well for angular measurements, by mounting the mirror D on the end of the arm whose deflections are to be measured (see Fig. 10). This arrangement, however, has the great disadvantage of altering the inclination of the mirror, and thus changing the width of the fringes at the same time that they are shifted in the field by the linear change in path.[2] This objection can be at once overcome by the modification shown in Fig. 11,

[1] Unfortunately Dr. Austin left the University before the experiments were completed.

[2] This same objection applies to the type of instrument first proposed for angular measurement by Michelson in his paper, Measurement by Light Waves, Amer. Journal, Feb., 1890, Plate III., Fig. 1; also Group III., Figs. 4 and 5.

in which the mirror B is mounted parallel to D, and on the same swinging arm which carries the latter. This is accomplished by introducing an additional mirror B', placed parallel to A and C. The effect of the angular motion of one mirror is compensated by the equal angular motion of the other, while if the point of suspension is between the two mirrors, as at O, the sensitiveness is doubled by the fact that a movement of the suspended system shortens one path at the same time that it lengthens the other. The only practical objection to the arrangement is that the mirrors B and D are not in the same plane, and are not therefore as easily mounted and managed as they would be if this were the case. The form in Fig. 12 accomplishes this last object by the use of the two additional mirrors, D' and D''.

The complete instrument as designed on these lines is shown in Fig. 12 *a*. It was built partly by Mr. Claflin, of the Claflin Dental Mfg. Co., of Worcester, and partly by Mr. Francis and myself in the machine shop of the university. Some features of construction differ from those since adopted as more desirable (see earlier part of this paper). For example, the fine adjustment of B (in this case applied to B') is accomplished by means of very fine watch screws, bearing directly against the mirror frame, and moved the required minute amount by long steel levers (one of which can be seen in the photograph) inserted in the heads of the pins.[1] In order to adapt the optical part of the instrument to suspended systems of different magnitude, the separating mirror A and mirror D' are mounted on one carriage, D'' on another, and B' on the third. These may be slid along to any part of the long V-shaped bed and clamped in any desired position. The mirror B' is mounted on a short auxiliary slide, and moved parallel to itself a small amount by a screw which takes the place of the screw which moves D in the linear type of instrument. To impart a slow motion to this screw, the nut in which the main screw takes its bearings is itself threaded, the pitch of this thread being very slightly greater than that of the screw. When this nut, therefore, is turned, as it may be when desired by the large milled head seen in the photograph, the screw is moved in or out at each revolution of the latter by an amount equal to the difference in the pitch of the thread on the outside of the nut and that on the inside. The screw itself is kept from revolving when the nut is turned by a friction collar which is not, however, tight enough to prevent its being turned directly by the small head when a rapid motion of B' is desired.

The whole bed may also be rotated about a heavy steel pivot at the center (by means of the very fine screw with large milled head seen at the back of the instrument), in order to bring these parts of the interferometer system parallel to the suspended system. The compensator C is mounted,

[1] The objections to this method have already been stated; see page 486.

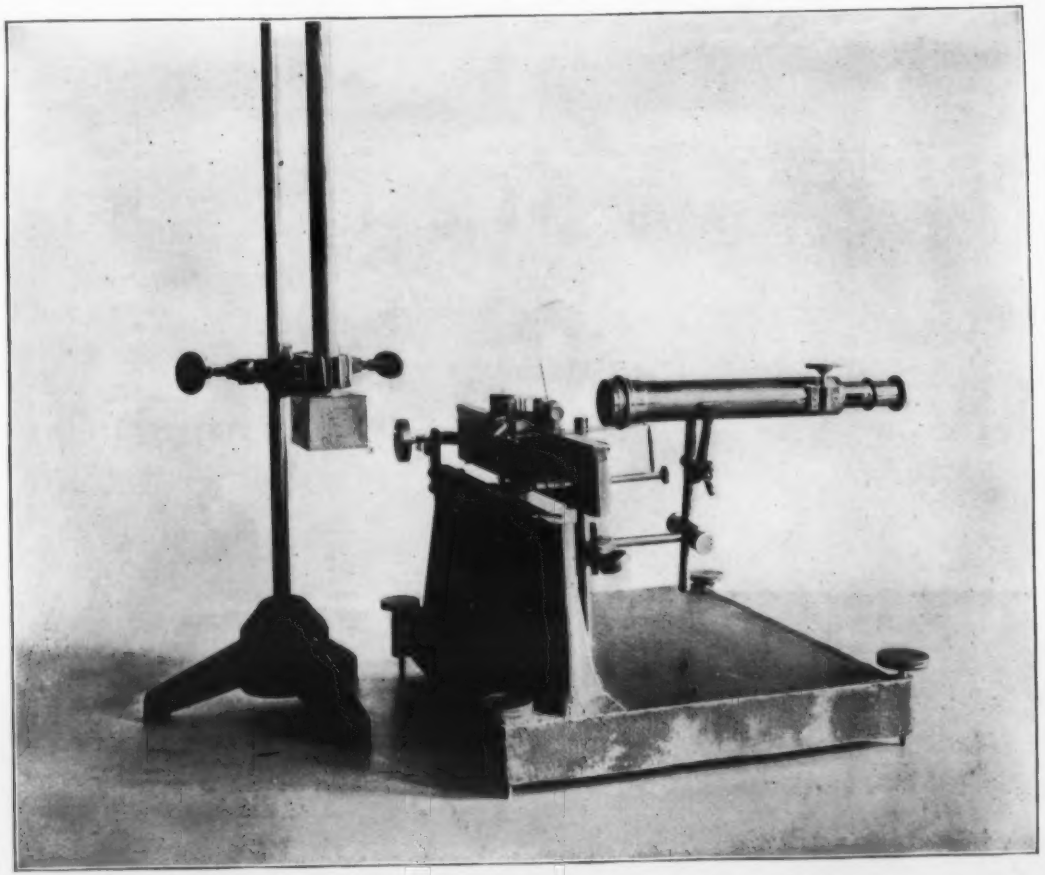


Fig. 12a.

not on the carriage with B', but just in front of it (between B and B' instead of between B' and A as in the diagrams in Figs. 2 and 12), on a heavy brass rod screwed into the base of the instrument. The spiral spring which twists this rod is attached, the one end to a light lever projecting from it, the other to the graduated sliding bar seen near the front. The strength of spring is so adjusted that a motion of this bar of from 15 to 20 cm. is necessary in order to twist the compensator glass sufficiently to introduce a difference of path of one wave length.

The suspended system consisted of the two small mirrors B, D, which were about 5 mm. square, mounted on a stiff though light arm made of glass fibers about 5 cm. long, and suspended at its center from a quartz fiber about 25 cm. long. It is of course impossible to adjust either of the suspended mirrors during the course of the experiment, and it was therefore necessary to be extremely careful in mounting them originally so that their surfaces lie very nearly in the same plane. This is done by laying them face down upon a piece of optical glass and examining the fringes between this surface and the faces of the mirrors.

The whole suspended system was inclosed in an air-tight chamber which consisted of a small cast-brass box with heavy walls, about 4 cm. square and 6 cm. long. The heavy brass tube[1] which carried the fiber was soldered into the top of the box and the front was closed by a thick plate of optical glass, cemented on with a mixture of ordinary glue and molasses. The top of the brass tube could also be closed by means of a brass cap which could be slipped over the head to which the upper end of the fiber was attached, and cemented in place after the latter had been adjusted. The tube and box were clamped in a heavy cast-iron stand, as shown in the plate, and the two parts of the instrument, the inclosed suspended system mounted as just described, and the optical "head," as it may be called, placed together on a very heavy stone slab, supported on rubber blocks. Blocks of stone, not shown in the plate, were also placed around the feet of the iron stand and under the brass box so as to steady the latter from below, as well as at the point where it was clamped above. It was an easy matter to obtain the fringes when the suspended system was held stationary, but it was only after many trials that they were obtained with a free system. The most convenient method of procedure was finally found to be as follows: The suspended system was first arrested in as nearly as possible its position of rest by means of a wire fork, mounted on a short brass rod passing through a hole in the side of the brass box, as shown in Fig. 13. After the fringes have been obtained in white light by the method already described, the system was gradually released by turning the rod by the long arm, b,

[1] A glass tube cemented into the box was first used, but afterward replaced by the metal as far more satisfactory.

outside the box, the bed being rotated by means of the tangent screw already spoken of, so as to follow the suspended system as it swings one way or the other, and keep the fringes in the field.[1] At first, as soon as the system was completely free, it began to swing and tremble so that it was very difficult to follow the fringes. The difficulties due to vibration were finally very considerably reduced, and at times completely overcome, by floating the stone carrying the whole apparatus in mercury, in a box very slightly larger than the stone itself, the stone being kept from touching the box by silk threads.[2] The observing telescope was also removed from the base of instrument and mounted on a separate stand,[3] so that after the preliminary adjustments had been made, nothing on the floating stone itself needed to be touched. The difficulties due to the small constant swinging of the system to and fro were largely overcome by proper damping. In the preliminary experimental work this was effected by attaching to the suspended system a long light bar magnet and placing strong bar magnets on the stone outside. Such an arrangement would not in general answer because of the directive force of the magnets and consequent loss of sensibility. Probably the best system of damping which would leave the sensibility unchanged would be that indicated by dotted lines in Fig. 13. It consists of two strips of copper, or better, pure silver foil, hung from the ends of the suspended system with the plane of the strips at right angles to the mirrors. These strips swing edgewise between the poles of two powerful horseshoe or electro magnets. The brass box containing the system was not large enough to allow this

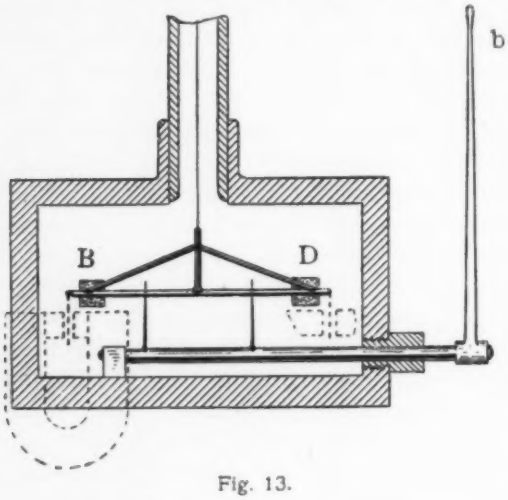


Fig. 13.

[1] Any motion of translation, or of rotation about a horizontal axis, of the system is of course without effect on the fringes.

[2] What seems to be an even better method of support to prevent vibration has recently been described by Professor Brace. See Astrophysical Journal, Vol. V., p. 214, March, 1897.

[3] The telescope may of course be placed at any desired distance from the instrument itself.

arrangement to be tried, but I feel no doubt that it would prove effective in holding the system at rest, or so nearly so that there would be no difficulty in noting the position of the fringes as they slowly swing to and fro. A considerable movement of the fringes is no disadvantage, — is in fact even an advantage, provided only the swing is steady and not so large as to throw the central white fringe (which is the only one which can be identified) out of the field. The zero point or position of rest is in such a case determined by the method of oscillations, just as in the case of the ordinary telescope and scale method of reading. To avoid parallax and other errors, the excursions of the central fringe are best noted by means of a scale engraved directly on one of the mirrors *D* or *B*.

These experiments having demonstrated the practicability of this method of reading deflections, I next began the construction of a large D'Arsonval galvanometer, designed with special reference to this new method of observation. Before this was finished, however, I was called upon to devote my whole time and energy to the design and construction of the large Paris interferometer, and immediately after its completion and installation in the laboratory of the International Bureau at Breteuil, I went to Washington to accept a position in the Smithsonian Astrophysical Observatory, where I had no opportunity to continue these experiments. It is only within the last few months, since my transfer to the Yerkes Observatory, that I have had an opportunity to resume the long-deferred completion of the galvanometer, the unfinished parts of which I have fortunately preserved.

It is as yet too soon, perhaps, to predict anything in regard to the degree of sensitiveness which it will be possible to practically attain with this instrument. A preliminary estimate, based on the dimensions of field-magnets and proposed coil and mirror system, would indicate that it would be possible to attain a sensitiveness such that a deflection of 0.1 fringe corresponds to a current of between 10^{-16} and 10^{-17} amperes. If this degree of sensitiveness can be attained, particularly with the D'Arsonval type of instrument, it will permit of certain problems of great theoretical interest being attacked for the first time experimentally.

There is one point of considerable importance in the theory of galvanometers in this connection. It might be urged by some, that this method of reading considerably increased the mass and, more particularly, the moment of inertia of the optical parts of the suspended system. This is true, but the remedy is simply to increase the mass of the rest of the suspended system in the same ratio. This is exactly the reverse of the modern practice of using very light systems to secure great sensitiveness; but as I have elsewhere[1] shown, this increased sensitiveness results not at all from

[1] Description of a Very Sensitive Form of Thomson Galvanometer and some Methods of Galvanometer Construction, Phil. Mag., Vol. 38, p. 553, December, 1894.

the lightness of the system, but from the higher degree of astaticism reached. Given the same degree of astaticism in the heavy system, and it would be the more sensitive of the two, although (and this is the practical difficulty generally attending the use of the heavy system) the time of swing will be much longer. With the interferometer method of reading, however, this slowness of swing is fortunately just what is wanted.

In conclusion it may be of interest to point out certain changes and modifications in the apparatus which have suggested themselves as desirable. In the first place, in making another instrument of this kind I would have all the mirrors larger, particularly the moving ones. In the instrument just described the mirrors A, B', C, D', and D'' were each about 10 mm. × 15 mm. and B and D, as already stated, about 5 mm. square. It would be better to have the first-named mirrors 5 mm. larger each way and the two last of about the same width as before, but at least twice as long, so as to allow greater amplitude of swing without causing the central fringe to pass out of the field.[1] The tangent screw which moves the bed in azimuth should be finer and act on the end of a longer arm, in order to be able to follow the movement of the mirror more slowly and steadily. It would be better for the same reason to have the steel pin upon which the bed turns, at one end of the latter instead of at the center. Both the bed and the mountings of the mirrors should be heavier (following more nearly the proportions given in the first part of this article) and the movable carriages on which the latter are mounted longer, and arranged to clamp more rigidly to the bed.

In the preceding form of instrument the difference in path Δ, introduced by a given angular movement θ, of the system is $2b\beta$, b being the distance BD between the centers of the mirrors. If we consider that the smallest movement that can be measured with certainty in a freely suspended system is one twentieth of a fringe,[2] the smallest angle that can be measured with the interferometer is

$$\theta_0 = \frac{1}{40}\frac{\lambda}{b}.$$

The smallest movement that can be observed with a mirror as ordinarily used is probably not more than one fifteenth its angular resolving power.[3]

[1] Or what amounts to the same thing, without allowing the point p, in Fig. 4, to get beyond the edge of the mirrors.

[2] With the type of instrument with fixed mirrors (linear measurement) a movement of one two-hundredth of a fringe may be detected under favorable conditions.

[3] See paper, "Manufacture of very Accurate Straight Edges," already referred to, p. 6, also review of Boys' determination of the Newtonian Constant of Gravitation, Astrophysical Journal, Vol. III., p. 303, April, 1896.

This, as is well known, is

$$\alpha = \frac{\lambda}{b},$$

and the smallest angular movement is therefore

$$\theta_m = \frac{1}{15}\frac{\lambda}{b}.$$

The interferometer system therefore is not so very much more accurate than a mirror of the same horizontal aperture used with the telescope and scale. The principal advantage of the former, as has been pointed out by Michelson, is that with it we need to use only two small pieces at the ends of a diameter instead of the whole mirror.[1] We could also do this in the case of the telescope without loss of resolving power provided we could make these two pieces form parts of one and the same optical surface, and also give sufficient light; but in general neither of these conditions could be fulfilled. When they can be, the interferometer does not offer sufficient increase in accuracy to compensate for the greater difficulties encountered in its use.[2] It occurred to me, however, during the course of these experiments, that we can further increase the attainable accuracy of the interferometer method by employing the principle of multiple reflection. One very simple and symmetrical arrangement which suggested itself is that shown in Fig. 14, in which, as before, B and D are the movable mirrors and F, F', F'' are three additional fixed mirrors. Each ray is reflected from the movable mirrors three times, and in such directions that the total difference in path produced by a given movement θ is now $6\,b\theta$, and the resolving power is therefore, on the same assumption as before,

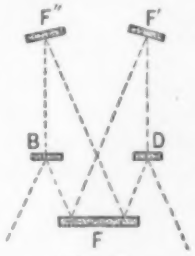


Fig. 14.

$$\theta_o = \frac{1}{120}\frac{\lambda}{b},$$

or eight times as great as for the mirror. But the greatest advantage of such a system is that we may, for a given resolving power, make the length of the arm supporting the two mirrors one third as great as in the more elementary form, and therefore reduce the amount of inertia to less than one tenth of what it was in the latter.

YERKES OBSERVATORY, UNIVERSITY OF CHICAGO,
March, 1897.

[1] This is an advantage only when the distance b is very large; at least fifteen times the height of the mirrors B and D.

[2] A fuller discussion of the comparative advantages of the different methods of observing small angular movements will be found in a paper, "On the Conditions which Determine the Ultimate Optical Efficiency of Methods for Observing Small Rotations," which will soon be published in the Phil. Mag.

SOME DETERMINATIONS OF THE SLIDE MODULUS OF GLASS, AND THE SHORTENING OF GLASS FIBERS WITH AGE.

BY W. S. FRANKLIN AND L. B. SPINNEY.

I. DETERMINATIONS OF SLIDE MODULUS.

Preparation of Fibers.

THE glass to be tested was drawn into fibers. Great care was taken to secure circular section and approximately uniform size, and the tension in drawing was carefully relieved as the glass began to harden. Some of the fibers were annealed, as indicated, by heating for about ten minutes to a temperature of about 750° C. in a long brass tube surrounded by a German silver resistance coil. To fasten the fibers in the torsion pendulum, the ends were platinized and imbedded in solder.

Measurement of Fibers.

For purposes of calculation the fibers were considered to be portions of two cones, and the diameter of each fiber was measured at each end and in the middle by means of a micrometer microscope, using a Zeiss stage micrometer as a reference standard. The lengths of fibers were measured in terms of a Brown & Sharpe steel scale.

Observations for slide modulus were taken in each case with a torsion pendulum made by suspending a body of known moment of inertia by the fiber to be tested.

Reduction Formula.

The formula for calculating slide modulus from the observation data was derived from the solution of the problem of the twisting of an acute cone. The authors are not aware of previous publication of this simple problem, and the outline of it is here given as applied to the present case.

Let $$r = ml \qquad \text{(i.)}$$

express the radius r of fiber from the near end to the middle; l is a coördinate parallel to the axis of the fiber and measured from the apex of the cone formed by this half of fiber, and m is a constant.

Similarly, $$r = m'l + a \qquad \text{(ii.)}$$

expresses the radius throughout the remainder of the fiber, m' and a being constants. Let l', l'', and l''' be the l coördinates of the near end, middle,

and far end of fiber, respectively, and r', r'', and r''' the radii of the fiber at these points. Then

$$\left.\begin{aligned} r' &= ml' \\ r'' &= ml'' \\ l'' - l' &= L \\ r'' &= m'l'' + a \\ r''' &= m'l''' + a \\ l''' - l'' &= L \end{aligned}\right\} \qquad \text{(iii.)}$$

in which L is the half length of the fiber. Each element of the fiber considered as a circular cylinder of radius r and length dl will be twisted through an angle

$$d\theta = \frac{2T}{\pi n} \cdot \frac{1}{r^4} \cdot dl,$$

by a torque T, n being the slide modulus. Therefore the angle of twist of the whole fiber is

$$\theta = \frac{2T}{\pi n}\int_{l'}^{l''} \frac{1}{m^4 l^4}\, dl + \frac{2T}{\pi n}\int_{l''}^{l'''} \frac{1}{(m'l + a)^4} \cdot dl. \qquad \text{(iv.)}$$

Substituting in this equation the values of m, m', a, l', l'', and l''' from (iii.), and remembering that the coefficient of T in (iv.) is equal to $\frac{t^2}{4\pi^2 K}$, in which t is the vibration period of a body of moment of inertia, K, suspended by the fiber, we have

$$n = \frac{2\pi KL}{3\, t^2 r'^3 r''^3 r'''^3}\left[r'''^3\,(r'^2 + r'r'' + r''^2) + r'^2\,(r''^2 + r''r''' + r'''^2)\right]. \qquad \text{(v.)}$$

This equation was used in the calculation of slide modulus from the observation data.

Results.

The results of tests of nine fibers are exhibited in the accompanying table. The wide variations in the value of the modulus cannot be ascribed to errors of observation, but it seems that the value of the modulus varies with the size of the fiber, and that it is changed by annealing the glass. German soft glass is not distinctly different from Jena normal thermometer glass.

II. Shortening of Fibers with Age.

The primary object of these tests was to determine the variation in length of extremely fine glass fibers with the hygrometric state of the surrounding air. This effect was at first confused with the shortening of the fibers with age, which entirely masked the effect sought. The apparatus was then used in the study of the shortening with age of several fibers,

and one of these, when twenty days old, was subjected to a rapid change of humidity up to saturation, without any distinct alteration in length. A change of one part in seven or eight millions could have been detected.

Fiber.	Glass.	Length $2L$.	Radii.	$n \times 10^{-11}$.
1	Jena ther. glass.	30.16	r' 0.003504 r'' 0.003920 r''' 0.004013	1.59
2	Jena ther. glass.	29.70	r' 0.006713 r'' 0.007378 r''' 0.007265	2.40
3	Jena ther. glass.	29.30	r' 0.002473 r'' 0.002389 r''' 0.002460	6.83
4	Jena ther. glass.	29.50	r' 0.003335 r'' 0.003435 r''' 0.004474	1.92
5	German soft glass.	29.30	r' 0.002777 r'' 0.002501 r''' 0.002496	2.08
6	German soft glass.	30.30	r' 0.002502 r'' 0.001865 r''' 0.002392	2.04
7	German soft glass.	28.60	r' 0.003860 r'' 0.003295 r''' 0.003323	1.77
9	German soft glass annealed.	29.50	r' 0.003555 r'' 0.003671 r''' 0.003852	3.63
10	German soft glass annealed.	29.20	r' 0.003433 r'' 0.003068 r''' 0.003740	2.93

Arrangement of Apparatus.

Two very fine glass fibers were stretched horizontally over a glass frame, and the ends of the fibers were bent and imbedded in solder. One fiber was tightly stretched and the other was loose, and was weighted at its middle with a bit of fine wire in the form of a hook. This glass frame was encased in an air-tight chamber, which was mounted, together with a micrometer microscope, upon a cast-iron plate. The micrometer

microscope served to measure the vertical distance between the middle points of the two fibers. In this way any changes in length of the loose fiber could be determined with great accuracy.

Results.

The effects of humidity could not be measured, as explained above. The accompanying diagram (Fig. 1) shows graphically the alterations in

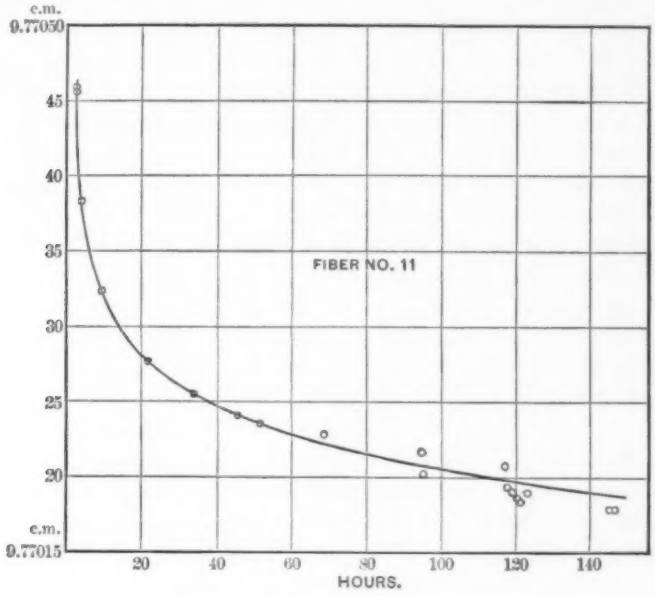


Fig. 1.

length of a very fine fiber of German soft glass. The first observation was taken about twenty minutes after the fiber was drawn. Jena thermometer glass gave a similar curve of shortening, and showed about the same percentage shortening in seven or eight days as German soft glass. The fibers were very fine, and the tension in drawing was, of course, not relieved as the glass began to harden. The fiber and frame were repeatedly removed from the case and replaced during the course of the observations from which the curve (Fig. 1) was derived, and the roughness of the curve is no doubt due to this fact.

AMES, IOWA, October 7, 1896.

THE MOST SENSITIVE ARRANGEMENT OF A WHEATSTONE BRIDGE WITH SPECIAL REFERENCE TO THE BOLOMETER.

BY C. D. CHILD AND O. M. STEWART.

THE usual theory of the Wheatstone bridge indicates that the most sensitive arrangement is secured when all of the resistances, including those of the galvanometer and battery circuits, are equal.[1] However, this theory assumes that we have at our command an invariable electromotive force. In reality, one is usually limited not by his inability to secure a larger electromotive force, but by the thermal effects in the bridge circuits when too large a current is used. This is especially the case in using a bolometer. When the current in the bolometer exceeds a certain maximum, excessive heating effects occur, and irregular movements of the galvanometer are observed. This maximum current is fixed entirely by the properties of the bolometer strips.

It has been the usual practice to work out the theory of the bolometer on the assumption that the electromotive force is constant, and then, after connecting up the bolometer in the manner indicated by this theory, to proceed to vary the electromotive force until a certain maximum current in the bolometer strips is reached. As this theory assumed a constant electromotive force, while in reality the electromotive force is varied, one is not warranted in concluding that this is the most sensitive arrangement. In fact it seems probable that for this maximum current there might be some other arrangement which would be more sensitive.

Thus we see that in solving the equations of the bolometer connections for maximum sensibility, the current in the strips should be treated as a constant, and the electromotive force as a variable. From this standpoint Dr. H. F. Reid[2] has shown that there are two solutions to the problem.

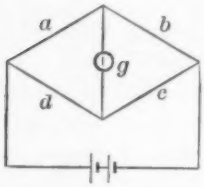


Fig. 1.

(1) If, in Fig. 1, a and b are the bolometer strips, the deflection of the galvanometer due to a small change in the resistance of the exposed strip is proportional to $\sqrt{\frac{1}{a+d}}$ and d and c should therefore be as small as possible. (2) If a and d are the bolometer strips, the deflection is proportional to $\sqrt{\frac{b}{a+b}}$, and, therefore, b and c should be as large as possible. As it has been found necessary to make the bolometer strips as nearly equal as possible, this equality was assumed in the derivation of the preceding results.

[1] Gray's Absolute Measurements, Vol. I., p. 331, and elsewhere.

[2] American Journal of Science, 135, p. 160.

For the first case, however, Dr. Reid concludes that if the resistances *d* and *c* are made small, the current through them would be large, and the heating effects would cause much trouble. And, therefore, as the gain in diminishing *d* and *c* is not large, the best working results will be obtained when all the resistances are made equal. In the second case, when *a* and *d* are the bolometer strips, he says that if the resistances *b* and *c* are made large, heating effects would again follow. This, as before, is to be avoided, and the resistances should again be made equal. This last conclusion we regard as incorrect. There is a difference between the effect of heat developed in a small resistance by a large current, and that developed in a large resistance by a small current. In the first case the heat is concentrated in a short wire, while in the second case it may be distributed over a long one.

Our attention was called to this matter while one of the writers was engaged on the work described elsewhere in this number of the PHYSICAL REVIEW, p. 433. In that work one of the bolometers used was first connected in the traditional way. Each of the arms, including the galvanometer, had a resistance of about one ohm. This arrangement was afterwards changed, and the connections made as in Fig. 1, where *a* and *d* represent the bolometer arms and *b* and *c* the other two arms, which were about 26 ohms each. The theory indicates that this change should produce a gain of about 30 per cent in the sensitiveness of the bolometer. The actual increase observed was in accordance with the predictions of this theory, and no increase in the unsteadiness of the galvanometer could be detected.

In order to verify the theory before reconstructing the bolometer connections, the following simple experiment was performed. The usual connections on a Wheatstone bridge were changed and others made as

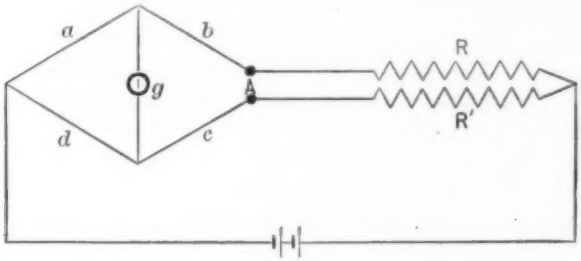


Fig. 2.

shown in Fig. 2. At *A* were two mercury cups which could be connected at will. The resistances *R* and *R'* were equal and were about 100 ohms each. The resistance in each of the arms *a*, *b*, *c*, *d* was one ohm. This arrangement fulfills the necessary condition of the theory, viz., that the

current in the arms a and a', which correspond to the bolometer strips, is constant. With connection made at A the usual arrangement of the bolometer connections is given, but when not made at that point we have the second case of Reid's theory. The resistances were carefully balanced, both when the connection at A was made and when it was broken. Then a slight increase was given to the resistance a', and the deflections of the galvanometer noted when connection was made at A and when it was broken. In accordance with the theory the deflection was greater in the latter case than in the former. In fact there seems to be no doubt but that a gain of about 30 per cent in the sensitiveness of a bolometer is secured by changing the resistance from the form commonly used to one where the resistance of the arms not used for bolometer strips is large and the corresponding increase made in the electromotive force.

Theory indicates that the galvanometer should have a resistance $\frac{2\,lR}{1+l}$ where R is the resistance of one of the bolometer strips and l the ratio between this resistance and that of one of the arms not a bolometer strip. Often it is not practicable to change the resistance of the galvanometer. But for this case also, theory shows that the arrangement of resistances suggested in this article is preferable.

PHYSICAL LABORATORY, CORNELL UNIVERSITY.

DEMONSTRATION OF THE DOPPLER EFFECT.

BY R. W. WOOD.

IN treating of the Doppler effect in the class room, it is usual to cite the example of the change in the pitch of a locomotive whistle in rapid motion. As a matter of fact scarcely one person in ten has any distinct recollection of ever having noticed the phenomenon, unless his attention has been directed to it, and it may be of interest to teachers to know that it can be shown by means of a very simple and inexpensive piece of apparatus.

A small pitch pipe such as is used by musicians is securely fastened to the end of a tapering stick six or seven feet long, such as a blackboard pointer. This can be blown with a piece of rubber tubing fastened along the rod, and by waving the device rapidly, it is possible to give the sounding pipe a to-and-fro velocity of twenty or thirty feet a second, which is sufficient to cause a marked rise and fall in the pitch. By waving the stick first in the direction of the "line-of-sight," and then at right angles to it, it can be shown that the effect occurs only in the former case. A bamboo fish pole with the partitions bored out, the pipe fastened into the tip by means of sealing wax, and a short piece of rubber hose inserted near the breach, would make a less clumsy piece of apparatus.

NOTE.

Galileo Ferraris. — Galileo Ferraris, physicist and engineer, died on the 7th of February, 1897. The city of Turin, where he met his sudden death from pneumonia, had been the central scene of his lifelong scientific activity. Although he was born at Livorno-Vercellese (1847), it was at Turin that he was educated; and that city has the honor of counting him among the illustrious of the graduates of her university (1867) and of her royal school of engineering.

In 1869, immediately upon the completion of his technical training, he was appointed assistant in applied physics in the *Museo Industriale* at Turin, and in 1879 he became professor of technical physics in that institution. The latter post he occupied until his death.

In the development of electrical engineering, which had its beginnings during this period, Ferraris was a pioneer. The laboratory, of which he was director, became one of the chief centers of activity in Italy in the domain of applied electricity. Ferraris was one of the first to investigate the currents generated in the telephone, and he was also one of the first to make practical experiments upon the transformer, and to establish the theory of that apparatus. In the development of the polyphase motor he was contemporaneous with Tesla, if indeed he did not anticipate the inventions of that electrician; and in this field, as in every subject upon which he wrote, his treatment was fundamental and exhaustive.

Ferraris' scientific activity, however, was by no means confined to the field of applied electricity. His contributions to optics, to mechanics, and to thermodynamics are scarcely less notable. His book on the *Cardinal Properties of Optical Instruments*, which appeared in 1878, is justly regarded as one of the most important treatises upon that subject. Every topic which he subjected to investigation he treated with that breadth and precision which characterizes the trained physicist, together with an insight into the purely practical and technical aspects of the subject which give his papers high rank from the standpoint of the engineer.

Ferraris' eminence as an electrician was fully recognized by his countrymen from the first. He was appointed by the government as an official delegate to the Electrical Congress in Paris in 1881, and he took part in the International Conference held in that city in 1882. At the Vienna exhibition, the following year, he again acted as delegate. Of the Turin exhibition of 1884 he was one of the chief organizers, and he was chosen

president of the section devoted to electricity. In 1886 he organized the Electrotechnical School for Engineers at Turin. This was the first school for electrical engineering established in Italy.

In one sense Ferraris' technical career may be said to have had its culmination at the Frankfort exhibition of 1891, where the principles of polyphase transmission, which he had worked out in so admirable a manner, were applied to the transmission of power from Lauffen to the exhibition grounds in Frankfort, a distance of over a hundred miles. Although this occasion may be regarded as a sort of triumphant climax, it did not mark the beginning of any decadence in Ferraris's activity, which indeed continued unabated up to the time of his sudden death. In the work of the Chamber of Delegates of the World's Congress of Electricians, held in Chicago in 1893, he took an important part.

During the very last year of his life Ferraris represented Piedmont as senator in the upper house of the Italian legislature.

E. L. N.

NEW BOOKS.

Molecules and the Molecular Theory of Matter. By A. D. RISTEEN. 8vo, pp. 223. Boston, Ginn & Co., 1895.

The experimental and theoretical developments of the past thirty years in molecular physics and chemical dynamics have almost completely transformed our views regarding the constitution of matter and created a new branch of natural science.

Although we have in some of the larger physical and chemical compendia elaborate discussions of certain phases of the subject, no exhaustive treatment of both the theoretical and experimental matter has yet appeared. The 1500-page compendium of Lehmann, or the treatise of Ostwald, illustrates the amount of material which can be selected out of the chaotic mass of matter which has accumulated in a comparatively short time.

Much, however, has been done in classifying facts and developing theory. Thomson's interesting lectures in physical and chemical dynamics suggest great possibilities in this line of development; and Boltzmann's *Gastheorie*, embodying Maxwell's theory, shows the finished groundwork for a dynamical theory in the simplest case.

The attempt to deduce thermodynamic laws from dynamical principles, initiated by Clausius and proved possible by Kirchhoff, and later perfected by von Helmholtz in his paper on "Least Action," and further supplemented by Boltzmann, has been crowned with success. Further extension in the same line has been made by Duhem and others, and indicates the present possibility in molecular dynamics.

The application of thermodynamics to chemical phenomena and electrical processes has received unusual attention during the past twenty years, and the work of Massieu, Gibbs, von Helmholtz, van't Hoff, and others has firmly established the true principles involved. With this link between chemical processes and dynamical principles, it is not a visionary possibility to expect that some day molecular phenomena will be as well established on dynamics as have Clausius, Maxwell, and Boltzmann established those of monatomic gases on the principles of Lagrange and Hamilton.

While the molecular theory has been so largely extended in this field, it has been sadly neglected in other lines. Little has been done on a molecular theory of sound. Such a theory might complicate rather than simplify this already difficult subject. However, many phenomena will not receive

their complete solution short of the use of such a theory. The propagation of very loud sounds of varying intensities may not be exactly determined with present methods, unless corrections depending on molecular considerations are introduced. In ordinary vibrations, thermodynamic considerations must enter, as instance the Laplacian correction to Newton's law as discussed and further corrected by Stokes.

In the subject of light, however, much has been done since the papers of Sellmeyer and von Helmholtz appeared. Kelvin's Baltimore lectures have done much to expand our views on the subject, and his gyrostatic molecule goes far toward a mechanical solution of magnetic rotary polarization, and serves also as a basis of an elastic solid theory.

Rayleigh has shown that our present laws in photometry are what should follow from the arbitrary motions in our kinetic theory, and also how the spectral lines are broadened from the same cause; and he has further extended the dynamics of diffraction of Stokes to the scattering of light by small particles. On the other hand, Kirchhoff has, by use of Green's theorem, extended Huyghens' principle, but he has, however, eliminated the idea of secondary or molecular disturbances assumed by Huyghens.

The complete agreement of von Helmholtz' conclusions on dispersion, both from the elastic solid and from the electromagnetic standpoint, with recent observations on both Röntgen rays and on the extreme ultra-red rays, strengthens the correctness of the assumptions; while the observed absorption by gratings of long heat waves indicates the true dynamical theory in doubly refracting substances.

The revival of Weber's theory of magnetism, with the adoption of a bipolar state of polarization in a dielectric by von Helmholtz, and the interesting application of Faraday tubes by Thomson, illustrate the activity in this branch of the subject. The experimental observations of the action of static charges in motion have done much to confirm, on the one hand, Weber's hypothesis of electrodynamic action in his famous controversy with von Helmholtz, and on the other, Maxwell's views as to dielectric polarization. This apparent anomaly in nature, of an action involving time as a variable, offers, perhaps, the only instance so far which is not reconcilable with the principle of the conservation of energy, as von Helmholtz has shown in his discussion.

The kinetic theory of gravitation has been much strengthened, and some of the inherent difficulties explained away.

Evidently a molecular theory for natural phenomena must mean much more than merely constitutions of the physicist's molecules or of the chemist's atoms in ponderable matter; we must premise other physical units differently endowed.

Any attempt to bring before the scientific student a brief résumé of

some of the present theories, and the facts upon which they are based, must certainly be appreciated. We have in the small volume before us an expansion of a popular lecture, which will be gladly welcomed by the beginner and commended by the teacher. However, the reader must content himself with only a brief reference to some of the better known phenomena and discussions. The author, however, has made an excellent selection for the purpose in view, and has presented most of it clearly and concisely. Several of the topics are interesting as a matter of history only, and might have been omitted for greater expansion on other points which have a living value in the theories of to-day, and which the author hardly carries far enough for the beginner.

The first part of the book is devoted to a short historical introduction on molecular constitution and the discussion of the kinetic theory of a gas treated as monatomic. Boltzmann's theorem of energy distribution, the explanation of pressure, temperature, specific heat, viscosity, law of molecular force, van der Waals' equation, etc., follow with appropriate examples. "Free path" and its bearing on high vacua receive some attention, inadequate to the extent and interest of the subject. Less exact, as it must be, is the next part on liquids. But much suggestive matter is given in regard to evaporation, ebullition, critical temperature, compressibility, etc., which, however, is largely speculation. Some classic experiments on capillarity are given, and an example of the action of molecular forces is explained in vaporization.

In the case of solids, the kinetic theory is illustrated rather briefly; but solutions, diffusion, osmotic pressure, crystallization, etc., are discussed and might well be expanded, considering the interest in this branch of physical chemistry. The discussion of the various methods of determining molecular magnitudes is more satisfactory and certainly interesting. Several pages are devoted to Dulong and Petit's law, the compound nature of the elements, the periodic law and the harmonics in atomic vibrations and the ultimate nature of matter.

The elastic solid theory is compared with the electromagnetic theory and rejected. This is unjust to the exponents of the contractile ether form of this theory, which, as proposed and worked out by Kelvin and extended to double refraction, dispersion, etc., by Glazebrook, explains as well as the electromagnetic theory, which is not entirely satisfactory. Duhem found on using von Helmholtz' earlier equations that there is the same difficulty in reflection as on the elastic solid theory. The difficulty at the ultimate bounds of such a medium is of the same order as that at the limits of an electrical medium containing a charge within it.

The author does not extend the molecular theory to include other physical units, but limits the discussion to the case of gravitation, to which several

pages are devoted in an interesting résumé of Kelvin and Maxwell's suggestions on the subject, omitting reference to more recent speculation "on the Continent."

Considering the scarcity of compiled material in English, the book is certainly a valuable addition to the popular literature of the subject.

D. B. BRACE.

A Manual of Physics. By WILLIAM PEDDIE. Second edition. 8vo, pp. xvii 573. New York, G. P. Putnam's Sons, 1896.

The first edition of this work has already been referred to in the pages of the PHYSICAL REVIEW.[1] In the new edition the general plan of the book has not been altered, although several portions have been rewritten. One of the most noticeable changes is in the mathematical methods used, which are considerably simplified. New material is added to the extent of nearly a hundred pages.

In the words of the author "this book is intended to be an accompaniment to a course of lectures on physics, by the use of which a student may largely avoid the evils of note-taking, and give more attention to the words of his teacher." Dr. Peddie's familiarity with the methods and requirements of the Scotch Universities makes it probable that the book is well suited to the needs of the Scotch and English student. Whether it will be equally useful in our American Universities is quite another question, and one which I should be inclined to answer in the negative. In this country a lecture course on general physics is usually largely experimental, and is intended to give to the student as concrete a notion as possible of the phenomena with which the science deals. To supplement such a lecture course a text-book is required which shall give a clear and rigorous statement of the underlying principles and theories. If descriptive matter can also be included, without making the book too bulky, this is doubtless desirable. In my opinion Dr. Peddie's *Manual* does not fulfill these requirements. While possessing many excellent features, the book is distinctly superficial in its treatment. The discussion of the second law of thermodynamics (p. 382) may be cited in support of this statement. The student who reads this discussion, and believes that he has learned something about the subject, will be in a worse position than one who has never studied it; for before proceeding further he will be compelled to disabuse his mind of many notions that are, to say the least, vague, and in some cases actually unsound.

While the superficial character of the book renders it unsuitable, in my

[1] Vol. I., p. 79.

opinion, as a text-book, a large amount of valuable matter is contained in Dr. Peddie's *Manual.* Many subjects are touched upon which are usually not mentioned in text-books at all; take, for example, the brief discussion of the various theories that have been suggested to explain gravitational attraction (p. 84). It would be a great help to the reader if references were given in such cases to some work where the subject is discussed in more detail. The tendency of the book is unquestionably to make one think, and for one who is well grounded in the fundamental principles of physics, so as to be on his guard against plausible but inexact methods of proof, the *Manual* will furnish interesting and suggestive reading.

ERNEST MERRITT.

Practical Work in Physics for Use in Schools and Colleges. Part III., Light and Sound. By W. G. WOOLLCOMBE. pp. x + 94. Oxford, The Clarendon Press, 1896.

This volume follows Part I., Practical Work in Heat, and Part II., Practical Work in General Physics.[1] The experiments are simple and suitable for beginners; no elaborate apparatus is required. The thirty experiments in light are classed in the following groups: Photometry, Reflection at Plane Surfaces, Reflection at Spherical Surfaces, Refraction at Plane Surfaces, Refraction through Lenses, Simple Optical Instruments. The twenty experiments in sound are grouped as follows: Transverse Vibrations of Wires, Velocity through Gases, Velocity through Solids, Interference. The directions are clear and the selection of material good, when viewed from the standpoint of the needs of elementary classes. B.

The Magnetic Circuit. By H. DU BOIS. Translated by Dr. ATKINSON. 8vo, pp. xviii + 362. London, Longmans, Green, & Co., 1896.

The translation of Dr. du Bois' work on the magnetic circuit makes available to the English-speaking public a treatment of the subject of magnetism that possesses several distinctive features. As I have already mentioned in discussing the German edition,[2] the book is not to be looked upon as one which will take the place of the well-known English works on magnetism. It will rather serve to fill some of the gaps in the treatment of the subject which have resulted from the special character of previous treatises. It is this characteristic of the work that has especially impressed me in using it as a book of reference.

[1] Reviewed in the PHYSICAL REVIEW, Vol. II., p. 317.
[2] PHYSICAL REVIEW, VOL. II., p. 311.

Even in those parts of the subject which must of necessity be common to all books on magnetism, the mode of presentation is sufficiently different from the usual one to offer valuable suggestions. In deriving the general laws of the magnetic field, for example, Dr. du Bois has discarded completely the conception of a magnetic pole. Strength of field is defined in terms of the electromotive force induced in a small test coil when the latter is rotated through 180°, and the whole system of equations in the theoretical part of the book is developed by reference to electro-dynamic actions and the phenomena of electro-magnetic induction. This method of approaching the subject possesses advantages that have already been recognized by several writers on the subject of magnetism. But I am not acquainted with any other work in which the method has been so systematically developed.

The work of translation is in general excellent. Numerous notes that have been added by Dr. du Bois, himself a master of the English language, make the translation more complete than the original.

Reference must be made to the review of the German edition for a more detailed discussion of the plan of the book.

ERNEST MERRITT.

A Primer of the Calculus. By E. SHERMAN GOULD. 12mo, pp. 91. Plates. New York, D. Van Nostrand Company, 1896.

This little book aims to give a working knowledge of the calculus where one independent variable and the first differential only are involved. The work is quite complete so far as it goes, and will prove useful in helping the reader to use the calculus as a tool in solving many of the problems in physics and mechanics.

INDEX TO VOLUME IV.

A.

Accumulators, Theory of, 353.
Absorption of the extraordinary ray in uniaxal crystals, *Stewart*, 433.
Admittance and impedance loci, *Bedell*, 143.
Alcohol, Note on the refractive index of, for electrical waves, *Mohler*, 153.
Refractive index and reflecting power of, for electrical waves, *Cole*, 50, 415.
Alternating current:
Circuits, Induction phenomena in, *Millis*, 128.
Electrolysis, Formation of lead sulphate in, *Sheldon and Waterman*, 324.
In parallel circuits with mutual induction, Division of, *Bedell*, 247.
Synchronous motor for determining the frequency of, *Moler*, 411.
American Association for the Advancement of Science, The Buffalo meeting of, 256.
Angular deflections of a suspended system, Application of the interferometer to the measurement of small, *Wadsworth*, 480.
Apparatus for illustrating the laws of falling bodies, *Randall and Markey*, 64.
Application of the interferometer to the measurement of small angular deflections of a suspended system, *Wadsworth*, 480.
Arno's method for studying dielectric hysteresis, 460, 463.
Atkinson, E., Translation of du Bois' *The Magnetic Circuit*, 511.
Atoms and molecules, Magnitude of, 215.

B.

Barus, C., On a possible development of the idiostatic electrometer, 400.
Bedell, F.:
Admittance and impedance loci, 143.
The division of alternating currents in parallel circuits with mutual induction, 247.
The Principles of the Transformer, 272.
Kapp's *Transformers for Single and Multiphase Currents*, 426.
Bolometer, The most sensitive arrangement of a Wheatstone bridge with special reference to, *Child and Stewart*, 502.
Book Reviews:
Alternating currents and alternating current machinery, *Jackson and Jackson*, 423.
Alternate current transformer, *Fleming*, 272.
American annual of photography and photographic times almanac, 432.
Chemistry at a glance: a study in molecular architecture, *Tuttle*, 432.
Electrical measurements: a laboratory manual, *Carhart and Patterson*, 265.
Elements of electrochemistry, *Le Blanc*, 432.
Grundriss der Electrochemie, *Jahn*, 76.
Higher mathematics: a text-book for classical and engineering colleges, *Merriman and Woodward*, 416.
Inorganic chemical preparations, *Thorp*, 432.
Laboratory manual of inorganic chemistry, *Williams*, 432.
Lehrbuch der Electrochemie, *Le Blanc*, 268.

Lehrbuch der Experimentalphysik, *Wüllner*, 261.
Magnetic circuit, *du Bois*, 511.
Manual of physics, *Peddie*, 510.
Mathematical papers read at the International Mathematical Congress, Chicago, 1893, *Moore, Bolza, Maschke, White*, 432.
Méthode et Principes des Sciences Naturelles, *Brentano*, 429.
Molecules and molecular theory of matter, *Risteen*, 507.
Motive power and gearing for electrical machinery, *Carter*, 430.
Physical measurements, *Weldon*, 352.
Physikalische-Chemische Propaedeutik, *Griesbach*, 351.
Physikalische Krystallographie, *Groth*, 71.
Physikalische Technik, *Frick*, 348.
Practical work in physics for use in schools and colleges, *Woolcombe*, 511.
Primer of quaternions, *Hathaway*, 421.
Primer of the calculus, *Gould*, 512.
Principles of physics, *Gage*, 159.
Principles of the transformer, *Bedell*, 272.
Problems and questions in physics, *Matthews and Shearer*, 432.
Spectralanalyse, *Landauer*, 158.
Submarine cable laying and repairing, *Wilkinson*, 351.
Theory of light, *Preston*, 79.
Theoretical chemistry, *Nernst*, tr. by Palmer, 55.
Transformers for single and multiphase currents, *Kapp*, 426.
Treatise on Bessel functions, *Gray and Mathews*, 67.
Brace, D. B., Risteen's *Molecules and Molecular Theory of Matter*, 507.
Brentano, Th. Funck, *Méthode et Principes des Sciences Naturelles*, 429.
British Association for the Advancement of Science, 258.
Bolza, Oskar, *Mathematical Papers*, 432.
Buckingham, E., Note on the theorem of Clausius, 39.
Palmer's translation of Nernst's *Theoretical Chemistry*, 155.
Buffalo Meeting of the American Association for the Advancement of Science, 256.

C.

Calibration of a thermometer, 217.
Capacity of condensers for short-charge periods, *Carpenter*, 238.
Carhart, Henry S., *Electrical Measurements: a Laboratory Manual*, 265.
Carpenter, H. V., A study of the apparent capacity of condensers for short-charge periods, 238.
Polarization and internal resistance of a galvanic cell, 329.
Carter, E. Tremlett, *Motive Power and Gearing for Electrical Machinery*, 430.
Child, C. D., The most sensitive arrangement of a Wheatstone bridge with special reference to the bolometer, 502.
Clark, J. A., The influence of a static charge of electricity on the surface tension of water, 375.
Clausius, Note on the theorem of, *Buckingham*, 39.
Cohn's method for the velocity of electrical waves, 51.
Cole, A. S., An example in thermometry, 217.
Cole, A. D., The refractive index and reflecting power of water and alcohol for electrical waves, 50, 415.
Condensers, Capacity of, for short-charge periods, *Carpenter*, 238.
Charge and discharge of, in circuits containing resistance, self-induction, and capacity, 128.
Oscillatory charge and discharge of, 138.
Conductivity of normal solutions, *Loomis*, 252.
Conversion of electric energy in dielectrics, *Threlfall*, 457.
Crystals, Absorption of the extraordinary ray in, *Stewart*, 433.

D.

D'Arsonval galvanometer read by interference fringes, 495.

Demonstration of the Doppler effect, *Wood*, 504.

Determinations of the slide modulus of glass, and the shortening of glass fibers with age, *Franklin and Spinney*, 498.

Dielectrics, Conversion of electric energy in, *Threlfall*, 457.

Dielectric hysteresis, Arno's method for studying, 460, 463.

Dielectrics, Viscosity of polarized, *Duff*, 23.

Dispersion formula, The Ketteler-Helmholtz, 312, 315.

Division of an alternating current in parallel circuits with mutual induction, *Bedell*, 247.

Doppler effect, Demonstration of the, *Wood*, 504.

du Bois, H., *The Magnetic Circuit*, 511.

Duff, A. Wilmer, Empirical formulæ for viscosity as a function of temperature, 404.

Viscosity of polarized dielectrics, 23.

Durand, W. F., Merriman and Woodward's *Higher Mathematics*, 416.

Note on different forms of the entropy function, 343.

Durgan, E. L., An example in thermometry, 217.

E.

Electric energy in dielectrics, Conversion of, *Threlfall*, 457.

Electric waves, Velocity of, *Saunders*, 81.

Visible, *Moore*, 149.

Electrical conductivity of normal solutions, *Loomis*, 252.

Electrical waves:

Refractive index and reflecting power of water and alcohol for, *Cole*, 50, 415.

Note on the refractive index of water and alcohol for, *Mohler*, 153.

Electricity, Mechanical conceptions of, *Franklin*, 388.

Electrolysis, Formation of lead sulphate in alternating current, *Sheldon and Waterman*, 324.

Electrolytic generator for oxygen and hydrogen, A new, *Franklin*, 61.

Electrometer:

New form of, 379.

On a possible development of the idiostatic, *Barus*, 400.

Energy measurements in the infra-red spectrum, Method for, *Nichols*, 297.

Entropy functions, Note on different forms of, *Durand*, 343.

Example in thermometry, *Cole and Durgan*, 217.

Expansion of metals, Measurement by the interferential method, *Morley and Rogers*, 1, 106.

Experimental determination of the temperature in Geissler tubes, *Wood*, 191.

Experimental study of induction phenomena in alternating current circuits, *Millis*, 128.

Extraordinary ray, Absorption of, in uniaxal crystals, *Stewart*, 433.

F.

Falling bodies, Apparatus for illustrating the laws of, *Randall and Markey*, 64.

Ferraris, Galileo, 505.

Fibers, Shortening of glass, with age, *Franklin and Spinney*, 498.

Fleming, J. A., *The Alternate Current Transformer*, 272.

Flow through capillary tube, Viscosity coefficient from, 209.

Formation of lead sulphate in alternating current electrolysis with lead electrodes, *Sheldon and Waterman*, 324.

Franklin, W. S.:

A new electrolytic generator for oxygen and hydrogen, 61.

Groth's *Physikalische Krystallographie*, 71.

On the mechanical conceptions of electricity and magnetism, 388.

Some determinations of the slide modulus of glass, and the shortening of glass fibers with age, 498.

Freezing-points of dilute aqueous solutions, *Loomis*, 273.

Frequency of an alternating current, Synchronous motor for determining the, *Moler*, 411.
Frick, F., *Physikalische Technik*, 348.

G.

Gage, A. P., *The Principles of Physics*, 159.
Galvanometer, Application of interference methods to the, 495.
Galvanic cell, Polarization and internal resistance of a, *Moore and Carpenter*, 329.
Geissler tubes, Experimental determination of the temperature in, *Wood*, 191.
Generator, A new electrolytic for oxygen and hydrogen, *Franklin*, 61.
Glass, Determinations of the slide modulus of, *Franklin and Spinney*, 498.
Glass fibers, Shortening of, with age, *Franklin and Spinney*, 498.
Goodwin, H. M.:
Note on the osmotic theory of the voltaic cell, 242.
The viscosity of mercury vapor, 207.
Gray, A., *A Treatise on Bessel Functions*, 67.
Griesbach, Dr. H., *Physikalische-Chemische Propaedeutik*, etc., 351.
Groth, P., *Physikalische Krystallographie*, etc., 71.
Gould, E. S., *A Primer of the Calculus*, 512
Gyroscopic pendulum, Trace of, *Merritt*, 336.

H.

Hathaway, A. S.:
A Primer of Quaternions, 421.
Gray and Mathews, *A Treatise on Bessel Functions*, 67.
Heat rays of great wave length, *Rubens and Nichols*, 314.
Howell, W. H., Griesbach's *Physikalische-Chemische Propaedeutik*, etc., 351.
Hydrogen, Electrolytic generator for, *Franklin*, 61.
Hysteresis, Arno's method for studying dielectric, 460, 463.

I.

Idiostatic electrometer, Possible development of, *Barus*, 400.
Impedance loci, *Bedell*, 143.
Index, Refractive, of water and alcohol for electrical waves, *Cole*, 50, 415.
Induction phenomena in alternating current circuits, *Millis*, 128.
Influence of a static charge of electricity on the surface tension of water, *Nichols and Clark*, 375.
Infra-red spectrum, A method for energy measurements in, *Nichols*, 297.
Interference of light method applied to the electrometer, 402.
Interferential method, Measurement of the expansion of metals by, *Morley and Rogers*, 1, 106.
Interferometer, Application to the measurement of small angular deflections of a suspended system, *Wadsworth*, 480.

J.

Jackson, D. C. and J. P., *Alternating Currents and Alternating-current Machinery*, 423.
Jahn, Hans, *Grundriss der Electrochemie*, 77.

K.

Kapp, Gisbert, *Transformers for Single and Multiphase Currents*, 426.
Kimball, A. S., Wüllner's *Lehrbuch der Experimentalphysik*, 261.
Klemenčič's thermopile resonator, 54.

L.

Landauer, Dr. John, *Die Spectralanalyse*, 158.
Laws of falling bodies, Apparatus for illustrating the, *Randall and Markey*, 64.
Lead Cell, *Moore*, 353.
Le Blanc, *Lehrbuch der Electrochemie*, 268.
Max, *The Elements of Electrochemistry*, 432.
Lecture-room demonstration of the orbits of bodies under central attraction, *Wood*, 413.

Loci, Admittance and impedance, *Bedell*, 143.
Loomis, E. H., On the specific gravity and electrical conductivity of normal solutions, 217.
The freezing-points of dilute aqueous solutions, 273.

M.

Macfarlane, Alexander, Hathaway's, *A Primer of Quaternions*, 421.
MacGregor, J. G., Carhart and Patterson's *Electrical Measurements*, 265.
Magnetism, Mechanical conceptions of, *Franklin*, 388.
Magnitude of atoms and molecules, 215.
Markey, W. H., An apparatus for illustrating the laws of falling bodies, 64.
Maschke, Heinrich, *Mathematical Papers*, 432.
Mathews, G. B., *A Treatise on Bessel Functions*, 67.
Matthews, Charles P., *Problems and Questions in Physics*, 432.
Measurement of the expansion of metals by the interferential method, *Morley and Rogers*, 1, 106.
Mechanical conceptions of electricity and magnetism, *Franklin*, 388.
Mercury vapor, The viscosity of, *Noyes and Goodwin*, 207.
Merriman, Mansfield, *Higher Mathematics*, 416.
Merritt, Ernest, Atkinson's du Bois' *The Magnetic Circuit*, 511.
Frick's *Physikalische Technik*, 348.
Gage's *The Principles of Physics*, 159.
Peddie's *A Manual of Physics*, 510.
The trace of the gyroscopic pendulum, 336.
Metals, Expansion of, measured by the interferential method, *Morley and Rogers*, 1, 106.
Method for energy measurements in the infra-red spectrum and the properties of the ordinary ray in quartz for waves of great wave length, *Nichols*, 297.
Meyer's formula for viscosity coefficient of a gas from the rate of flow through a capillary tube, 209.
Millis, F. E., An experimental study of induction phenomena in alternating current circuits, 128.
Mirrors, Silvering of, 487.
Modulus of glass, Slide, *Franklin and Spinney*, 498.
Mohler, John F., Note on the refractive index of water and alcohol for electrical waves, 153.
Molecules, Magnitude of, 215.
Moler, G. S., A synchronous motor for determining the frequency of an alternating current, 411.
Moore, B. E., Jahn's *Grundriss der Electrochemie*, 78.
Le Blanc's *Lehrbuch der Electrochemie*, 268.
Polarization and internal resistance of a galvanic cell, 329.
The lead cell, 353.
Visible electric waves, 149.
Moore, E. Hastings, *Mathematical Papers*, 432.
Morley, E. W., On the measurement of the expansion of metals by the interferential method, 1, 106.
Mutual induction, Division of an alternating current in circuits with, *Bedell*, 247.

N.

Nernst, W., *Theoretical Chemistry*, 155.
Nichols, E. F.:
A method for energy measurements in the infra-red spectrum and the properties of the ordinary ray in quartz for waves of great wave length, 297.
Heat rays of great wave length, 314.
Nichols, E. L:
Landauer's *Die Spectralanalyse*, 158.
The influence of a static charge of electricity on the surface tension of water, 375.
Note on different forms of entropy function, *Durand*, 343.
Note on the osmotic theory of the voltaic cell, *Goodwin*, 242.

Note on the refractive index of water and alcohol for electrical waves, *Mohler*, 153.
Note on the theorem of Clausius, *Buckingham*, 39.
Noyes, A. A., The viscosity of mercury vapor, 207.

O.

Optical principles of the interferential method, 3.
Orbits of bodies under the action of a central attraction, Demonstration of, *Wood*, 413.
Ordinary ray in quartz, *Nichols*, 297.
Oscillatory charge and discharge of condensers, 138.
Osterberg, Max, Synopsis of current electrical literature for 1895, 80.
Oxygen, Electrolytic generator for, *Franklin*, 61.

P.

Palmer, C. S., Nernst's *Theoretical Chemistry*, 155.
Parallel circuits with mutual induction, Division of an alternating current in, *Bedell*, 247.
Patterson, G. W., *Electrical Measurements: a Laboratory Manual*, 265.
Pendulum, Gyroscopic, *Merritt*, 336.
Peddie, W., *A Manual of Physics*, 510.
Platt, C., Brentano's *Méthode et Principes des Sciences Naturelles*, 429.
Polarization and internal resistance of a galvanic cell, *Moore and Carpenter*, 329.
Polarized dielectrics, Viscosity of, *Duff*, 23.
Polarizer, New form of, 443.
Pressure, Influence on thermometer readings, 229.
Preston, T., *The Theory of Light*, 79.

R.

Radiometer:
As a measuring instrument, 300.
Construction of a new form of, 298.
Randall, H. M., An apparatus for illustrating the laws of falling bodies, 64.
Reflecting power of a metal for electrical waves, 55.
Of water and alcohol for electrical waves, *Cole*, 50, 415.
Reflection by fluorite, 301.
By quartz, 303.
Of electrical oscillations by liquid surfaces, 57.
Formula for index of, 310.
Refractive index and reflecting power of water and alcohol for electrical waves, *Cole*, 50, 415.
Index of water and alcohol for electrical waves, Note on, *Mohler*, 153.
Refractometer method applied to the electrometer, 402.
Resistance of a galvanic cell, *Moore and Carpenter*, 329.
Resonator, thermopile, 54.
Risteen, A. D., *Molecules and the Molecular Theory of Matter*, 507.
Rogers, W. A., On the measurement of the expansion of metals by the interferential method, 1, 106.
Rubens, H., Heat rays of great wave length, 314.

S.

Saunders, C. A., The velocity of electric waves, 81.
Shearer, J. S.:
Preston's *Theory of Light*, 79.
Problems and Questions in Physics, 432.
Sheldon, S., On the formation of lead sulphate in alternating current electrolysis with lead electrodes, 324.
Silvering of mirrors, 487.
Specific gravity of normal solutions, *Loomis*, 252.
heats of the metals, *Waterman*, 161.
Spectrum, Method for energy measurements in the infra-red, *Nichols*, 297.
Sphere, Formula for the descent of, through a viscous liquid, 25.
Spinney, L. B., Some determinations of the slide modulus of glass, and the shortening of glass fibers with age, 498.
Steinmetz, C. P., Jackson's *Alternating Currents and Alternating-current Machinery*, 423.

Stewart, O. M.:
On the absorption of the extraordinary ray in uniaxal crystals, 433.
The most sensitive arrangement of a Wheatstone bridge with special reference to the bolometer, 502.
Stokes' formula for the descent of a sphere through a viscous liquid, 25.
Storage-battery, Theory of, 353.
Stratified discharge in Geissler tubes, 201.
Study of the apparent capacity of condensers for short-charge periods, *Carpenter*, 238.
Surface tension of water, Influence of a static charge of electricity on, *Nichols and Clark*, 375.
Suspended system, Application of the interferometer to the measurement of small angular deflections of, *Wadsworth*, 480.
Synchronous motor for determining the frequency of an alternating current, *Moler*, 411.
Synopsis of current electrical literature for 1895, *Osterberg*, 80.

T.

Temperature in Geissler tubes, Experimental determination of, *Wood*, 191.
Temperature of drops of water determined, 377.
Temperature, Viscosity as a function of the, *Duff*, 404.
Theorem of Clausius, Note on, *Buckingham*, 39.
Theory of the voltaic cell, Note on the osmotic, *Goodwin*, 242.
Threlfall, Richard, On the conversion of electric energy in dielectrics, 457.
Thermometer, Calibration of, 217.
Correction for pressure, 229.
Fundamental interval of, 232.
Thermometry, An example in, *Cole and Durgan*, 217.
Thorp, Frank Hall, *Inorganic Chemical Preparations*, 432.
Thurston, R. H., Carter's *Motive Power and Gearing for Electrical Machinery*, 430.
Trace of the gyroscopic pendulum, *Merritt*, 336.
Transformer, Effect of magnetic leakage, 148.
Effect of the variation of the external secondary self-induction, 147.
Primary current locus, 147.
Tuttle, Herbert B., *Chemistry at a Glance*, 432.

U.

Uniaxal crystals, Absorption of the extraordinary ray in, *Stewart*, 433.

V.

Velocity of electrical waves, *Saunders*, 81.
Effect of the resistance of the wires on, 99.
Velocity of long electrical waves, 51.
Visible electric waves, *Moore*, 149.
Viscosity as a function of temperature, Empirical formulæ for, *Duff*, 404.
Viscosity of mercury vapor, *Noyes and Goodwin*, 207.
Viscosity of polarized dielectrics, *Duff*, 23.
Viscous liquid, Formula for the descent of a sphere through, 25.
Voltaic cell, Note on the osmotic theory of, *Goodwin*, 242.

W.

Wadsworth, F. L. O., The application of the interferometer to the measurement of small angular deflections of a suspended system, 480.
Water, Note on the refractive index of, for electrical waves, *Mohler*, 153.
Refractive index and reflecting power of, for electrical waves, *Cole*, 50, 415.
Waterman, F. H., The specific heats of the metals, 161.
Waterman, Marcus B., On the formation of lead sulphate in alternating current electrolysis with lead electrodes, 324.
Wave length, Heat rays of great, *Rubens and Nichols*, 314.
Waves, Note on the refractive index of water and alcohol for electrical, *Mohler*, 153.

Waves of great wave length, Properties of the ordinary ray in quartz for, *Nichols*, 297.

Waves, Refractive index and reflecting power of water and alcohol for electrical, *Cole*, 50, 415.

Velocity of electric, *Saunders*, 81.

Visible electric, *Moore*, 149.

Weldon, Frank C., *Physical Measurements*, 352.

Wheatstone bridge, Most sensitive arrangement of, with special reference to the bolometer, *Child and Stewart*, 502.

White, Henry S., *Mathematical Papers*, 432.

Wilkinson, H. D., *Submarine Cable Laying and Repairing*, 351.

Williams, Rufus P., *Laboratory Manual of Inorganic Chemistry*, 432.

Wood, R. W., Demonstration of the Doppler effect, 504.

Lecture-room demonstration of the orbits of bodies under the action of a central attraction, 413.

Woodward, Robert S., *Higher Mathematics*, 416.

Woolcombe, W. G., *Practical Work in Physics*, 511.

Wüllner, Adolph, *Lehrbuch der Experimentalphysik*, 261.

VOLUME IV NUMBER 5

THE PHYSICAL REVIEW

A JOURNAL OF EXPERIMENTAL AND THEORETICAL PHYSICS

CONDUCTED BY

EDWARD L. NICHOLS

ERNEST MERRITT, and FREDERICK BEDELL

XXIII

MARCH–APRIL 1897

TABLE OF CONTENTS

The Lead Cell E. E. MOORE 353

The Influence of a Static Charge of Electricity upon the Surface Tension of Water E. L. NICHOLS and J. A. CLARK 375

On the Mechanical Conceptions of Electricity and Magnetism W. S. FRANKLIN 383

Minor Contributions: (1) On a Possible Development of the Idiostatic Electrometer. *C. Barus.* (2) Empirical Formulæ for Viscosity as a Function of Temperature. *A. Wilmer Duff.* (3) A Synchronous Motor for Determining the Frequency of an Alternating Current. *G. S. Moler.* (4) Lecture-room Demonstration of Orbits of Bodies under the Action of a Central Attraction. *R. W. Wood.* (5) The Refractive Index of Water and Alcohol for Electrical Waves. *A. D. Cole* 400

New Books: *Merriman and Woodward:* Higher Mathematics. *Hathaway:* A Primer of Quaternions. *D. C. and J. P. Jackson:* Alternating-Current Machinery. *Kapp:* Transformers for Single and Multiphase Currents. *Brentano:* Méthode et Principes des Sciences Naturelles. *Carter:* Motive Power and Gearing for Electrical Machinery. *Le Blanc:* Elements of Electrochemistry. Mathematical Papers read at the International Mathematical Congress, Chicago, 1893. *Williams:* Laboratory Manual of Inorganic Chemistry. *Tuttle:* Chemistry at a Glance. *Thorp:* Inorganic Chemical Preparations. The American Annual of Photography. *Matthews and Shearer:* Problems and Questions in Physics 416

COPYRIGHT, 1897, BY THE MACMILLAN COMPANY

PUBLISHED FOR CORNELL UNIVERSITY

NEW YORK: THE MACMILLAN COMPANY

LONDON: MACMILLAN AND CO., LTD.

BERLIN: MAYER AND MUELLER

THE PHYSICAL REVIEW.

A JOURNAL OF EXPERIMENTAL AND THEORETICAL PHYSICS.

CONDUCTED BY

EDWARD L. NICHOLS,

ERNEST MERRITT, AND FREDERICK BEDELL.

THE PHYSICAL REVIEW is published bi-monthly, each annual volume beginning with the July–August number. The price of subscription is three dollars a year, or fifty cents a number. Subscriptions and applications for advertising space should be sent to the publishers, Messrs. MACMILLAN & Co., *New York and London;* or, to Messrs. MAYER AND MUELLER, *Berlin.*

Subscribers who desire to have their copies of THE PHYSICAL REVIEW sent to them with the edges cut, can so obtain them by informing the publishers to that effect.

Correspondence relating to contributions should be addressed to the editors, at *Ithaca, New York.*

Manuscript intended for publication in THE PHYSICAL REVIEW must be communicated by the author; when publication in other journals is contemplated, notice to this effect should be given.

The authors of original articles published in the REVIEW will receive one hundred separate copies in covers, for which no charge will be made; additional copies, when ordered in advance, may be obtained at cost.

PUBLISHED FOR CORNELL UNIVERSITY.

THE MACMILLAN COMPANY, NEW YORK.

TEXT-BOOKS ON GENERAL PHYSICS

AND DIFFERENT BRANCHES OF PHYSICAL SCIENCE.

JUST READY.

The Outlines of Physics.

AN ELEMENTARY TEXT-BOOK.

By EDWARD L. NICHOLS,

Professor of Physics in Cornell University.

12mo. Cloth. pp. xi + 452. Price $1.40, net.

The important principles of Physics are stated in as clear and simple a manner as possible, with an abundance of laboratory exercises. The disciplinary value of the course is comparable to that of a year of mathematics required for entrance to college.

A Text-Book of the Principles of Physics.

By ALFRED DANIELL, D.Sc.

Illustrated. Cloth. 8vo. New revised edition, $4.00, net.

A recognized standard wherever the English language is understood, and withal one of the most readable works on Physics among those not intended for popular use. — *Scientific American.*

An excellent book of reference. — *The Nation.*

Its accuracy and reliability are now nearly as perfect as can be hoped for. — *Journal of Education.*

BY THE SAME AUTHOR.

Physics for Students of Medicine.

16mo. Cloth. pp. xv + 469. Price $1.25, net.

CONTENTS.

Units of Measurement. — The C. G. S. System.
Motion of Bodies. — Degrees of Freedom, Translation, Rotation, Deformations or Strains, Oscillations or Vibrations.
Friction. — Statical, Kinetical, Work.
Matter. — Properties, Essential and General, Ether, Molecules and Atoms, Gases, Liquids, Solids.
Sound. — Production and Propagation.
Heat. — A Form of Energy, Temperature, Conduction and Convection, Transformation of Heat into Work.
Ether-Waves. — Production, Reflexion and Refraction, Lenses, Optical Instruments, Interference, Double Refraction.
Electricity. — Galvanic Cells, Effects of a Current, Quantity of Electricity, Electrostatic Induction, Production of Difference of Potential, the Variable Period, Alternating Currents, Electro-magnetic Waves.

THE MACMILLAN COMPANY.

NEW YORK: 66 FIFTH AVENUE.
CHICAGO: ROOM 23, AUDITORIUM.
SAN FRANCISCO: 327-331 SANSOME STREET.

Alternating Currents

AND

Alternating Current Machinery.

BY

DUGALD C. JACKSON, C.E.,

Professor of Electrical Engineering in the University of Wisconsin; Member of the American Society of Mechanical Engineers, the American Institute of Electrical Engineers, etc.;

AND

JOHN P. JACKSON, M.E.,

Professor of Electrical Engineering in the Pennsylvania State College; Member of the American Institute of Electrical Engineers, etc.

12mo. Cloth. pp. xvii + 729. Price $3.50, net.

COMMENTS.

"It appears to be a very valuable work, and I have no doubt it will be thoroughly appreciated by practical electrical engineers as well as instructors." — RALPH W. POPE, *Secretary Amer. Inst. of Elec. Engineers.*

"I like the way in which they develop their theory and make practical applications. I feel sure that the book will receive a hearty welcome." — GEORGE W. PATTERSON, JR., *Asst. Prof. of Physics, University of Michigan.*

"We believe this is the only text-book on the subject worthy of the name; and it is, in our opinion, a matter of congratulation that the student has at hand at last something adapted to his needs." — *The Electrical Engineer*, New York.

"I have taken occasion to read with some care the work on 'Alternating Currents and Alternating Current Machinery' by Dugald C. and J. P. Jackson. It better fulfils, I think, the requirements of a text-book for students in electrical engineering than any work with which I am acquainted. I find especially valuable the chapters devoted to the discussion of different arrangements of resistance, capacity, and inductance in circuits, both series and parallel, matter which in this work is most clearly treated.

"Then, too, I conceive it to be a matter of considerable consequence that some discussion be given of the use of complex quantities in alternating current problems as is here done. . . .

"The general appearance of the work is excellent. I am using it with my fourth-year students in electrical engineering, who find it a most helpful work."

H. E. CLIFFORD, S.B., *Asst. Prof. of Theoretical Physics, Mass. Inst. of Tech., Boston.*

THE MACMILLAN COMPANY,

NEW YORK: 66 FIFTH AVENUE.
CHICAGO: ROOM 23, AUDITORIUM.
SAN FRANCISCO: 327-331 SANSOME STREET.

THE PRINCIPLES OF THE TRANSFORMER.

By FREDERICK BEDELL, Ph.D.,

Assistant Professor of Physics in Cornell University.

8vo. Cloth. pp. ix + 416. Price $3.25 net.
250 Illustrations.

THIS work constitutes a systematic treatise on the Alternating Current Transformer, and a logical exposition of the principles involved, of the most modern methods of transformer design, construction, and testing, and of transformer systems of distribution. Portions of the book were originally prepared with the collaboration of Dr. A. C. Crehore. The work is suited for practical engineers and for students in electrical engineering.

AMONG THE CHAPTERS ARE THE FOLLOWING:

TRANSFORMER SYSTEMS OF DISTRIBUTION.
THE MAGNETIC CIRCUIT OF THE TRANSFORMER.
THE ALTERNATING CURRENT.
THE TRANSFORMER DIAGRAM.
SIMPLE THEORY OF THE TRANSFORMER.
CONSTANT CURRENT TRANSFORMER.
CONSTANT POTENTIAL TRANSFORMER.
DESIGN AND CONSTRUCTION.
EXPERIMENTAL TRANSFORMER DIAGRAMS.
INSTANTANEOUS TRANSFORMER CURVES.
TRANSFORMER TESTING.
EFFECT OF HYSTERESIS AND FOUCAULT CURRENTS.

OPINIONS OF THE PRESS.

To gather all this into one book, and to formulate a definite and intelligible scheme out of the multifarious and often contradictory material at hand, was not an easy task, but the author has entirely succeeded. To the student and to the practical maker of transformers the work is invaluable. — *The Automotor.*

The same clearness of reasoning and lucidity of style which has justly rendered the previous work [Bedell and Crehore's *Alternating Currents*] popular among students are apparent in this volume also. — *American Journal of Science.*

Before its appearance the student was compelled to rely upon books which were illogical collections . . . hastily thrown together in book form. [One] should recognize the endeavor of Professor Bedell to bring order out of chaos in presenting the fundamental equations . . . in such a clear and instructive manner. — PROFESSOR TROWBRIDGE, in *The Electrician.*

While professedly exponent of the principles of the alternating current transformer, the author also deals with the practical sides of the question, both as regards manufacture and use. This part of the subject occupies the latter half of the volume; . . . the work should be of as much value to the English as to the American electrician. — *Electricity*, London.

The work is interesting and instructive, the style is very clear, and the various theories impartially examined, some good experimental diagrams are given, also useful information on testing transformers. — *Electrical Review*, London.

. . . The special treatment of several problems is decidedly new. The author has not confined himself to the mathematical and graphical treatment of the subject, . . . but has incorporated many useful hints on the practical side of the subject. It is a valuable addition to the literature on transformers. — *The Electrical Engineer*, London.

THE MACMILLAN COMPANY,

NEW YORK: 66 FIFTH AVENUE.
CHICAGO: ROOM 23, AUDITORIUM.
SAN FRANCISCO: 327-331 SANSOME STREET.

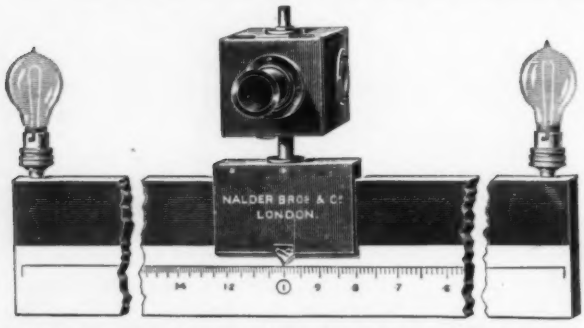
NALDER BROS. & CO.,

LONDON,

ENGLAND.

Cable Address: SECOHM, LONDON. A 1 and A.B.C. Codes used.

NO AGENTS IN THE UNITED STATES.



SCIENTIFIC APPARATUS.

N.C.S. PATTERN OF THE TROTTER BAR PHOTOMETER.

This is the instrument selected by the English General Post Office for their tests on incandescent lamps. B. A. Meeting, 1896. (See *Electrician*, Feb. 10, 1896.)

Much easier to read than either the Shadow or Grease spot instrument.

Complete, with direct reading scale, ready to mount on bar.

Price $32.00.

ELECTRICAL INSTRUMENTS.

AYRTON MATHER UNIVERSAL SHUNT, NEW PATTERN.

Does not require to be adjusted to the resistance of the instrument, but gives 1, 3, 10, 30, 100, 300, and 1000 multiplying powers, irrespective of galvanometer resistance.

Price $16.00.

SPECIAL TERMS TO EDUCATIONAL INSTITUTIONS.

WRITE FOR CATALOGUE OF INSTRUMENTS.

A Tonic

For Brain-Workers, the Weak and Debilitated.

Horsford's Acid Phosphate is, without exception, the Best Remedy for relieving Mental and Nervous Exhaustion; and where the system has become debilitated by disease, it acts as a general tonic and vitalizer, affording sustenance to both brain and body.

Dr. E. Cornell Esten, Philadelphia, Pa., says: "I have met with the greatest and most satisfactory results in dyspepsia and general derangement of the cerebral and nervous systems, causing debility and exhaustion."

Descriptive pamphlet free on application to

Rumford Chemical Works, Providence, R.I.

Beware of Substitutes and Imitations.

For sale by all Druggists.

THE TWO GREAT SCIENTIFIC JOURNALS.

SCIENCE,

A Weekly Journal devoted to the Advancement of Science.

Editorial Committee: S. Newcomb, *Mathematics;* R. S. Woodward, *Mechanics;* E. C. Pickering, *Astronomy;* T. C. Mendenhall, *Physics;* R. H. Thurston, *Engineering;* Ira Remsen, *Chemistry;* J. Le Conte, *Geology;* W. M. Davis, *Physiography;* O. C. Marsh, *Paleontology;* W. K. Brooks, *Invertebrate Zoology;* C. Hart Merriam, *Vertebrate Zoology;* S. H. Scudder, *Entomology;* N. L. Britton, *Botany;* Henry F. Osborn, *General Biology;* H. P. Bowditch, *Physiology;* J. S. Billings, *Hygiene;* J. McKeen Cattell, *Psychology;* Daniel G. Brinton, J. W. Powell, *Anthropology.*

Single Copies, 15c.; Annual Subscription, $5.

NATURE,

A Weekly Illustrated Journal of Science.

Single Copies, 15c.; Annual Subscription, $6.

SPECIAL OFFER.

To those not already subscribers to *Science*, we will send both *Science* and *Nature*, postage prepaid, **on receipt of $10.00.**

THE MACMILLAN COMPANY, Publishers,
66 FIFTH AVENUE, NEW YORK.

PEERLESS.

"True Merit Wins Success."

It has every good quality, every improvement, and every advantage known to any other machine, with many that they have not. It will do more of the best work with least labor, and will do that work for a greater length of time.

Arrangements for their sale desired in every city on the globe where not now represented. Send for descriptive circular.

PEERLESS TYPEWRITER CO.,

Principal Office and Factory, - - - **ITHACA, N.Y., U.S.A.**

6m

PUBLISHED BY THE MACMILLAN COMPANY.

Problems and Questions in Physics.

By CHARLES P. MATTHEWS, M.E.,
Associate Professor of Electrical Engineering, Purdue University, formerly Instructor in Physics, Cornell University,
AND
JOHN SHEARER, B.S.,
Instructor in Physics, Cornell University.
8vo. Cloth. pp. 247 + 4. Price $1.60, net.

An Intermediate Course of Practical Physics.

BY ARTHUR SCHUSTER, Ph.D., F.R.S.,
Langworthy Professor of Physics and Director of the Physical Laboratory in the Owens College, Manchester,
AND
CHARLES H. LEES, D.Sc.,
Senior Assistant Lecturer and Demonstrator in Physics in the Owens College, Manchester.
MACMILLAN'S MANUALS FOR STUDENTS.
12mo. Cloth. pp. xv + 248. Price $1.10, net.

A Laboratory Manual of Physics and Applied Electricity,

Arranged and Edited by EDWARD L. NICHOLS,
Professor of Physics in Cornell University.
IN TWO VOLUMES.
Vol. I. Cloth. Price $3.00, net.
JUNIOR COURSE IN GENERAL PHYSICS. By ERNEST MERRITT and FREDERICK J. ROGERS.
Vol. II. Cloth. pp. 444. Price $3.25, net.
SENIOR COURSES AND OUTLINE OF ADVANCED WORK. By GEORGE S. MOLER, FREDERICK BEDELL, HOMER S. HOTCHKISS, CHARLES P. MATTHEWS, and THE EDITOR.

A large proportion of the students for whom primarily this Manual is intended are preparing to become engineers, and special attention has been devoted to the needs of that class of readers. In Vol. II., especially, a considerable amount of work in applied electricity, in photometry, and in heat has been introduced.

NOW READY. Vol. II. SOUND AND LIGHT. 8vo. Price, $1.50, *net.*

The Elements of Physics.

By EDWARD L. NICHOLS, B.S., Ph.D.,
Professor of Physics in Cornell University,
AND
WILLIAM S. FRANKLIN, M.S.,
Professor of Physics and Electrical Engineering at the Iowa Agricultural College, Ames, Ia.
PART I. In Three Volumes: Vol. I. Mechanics and Heat; Vol. II. Electricity and Magnetism; Vol. III. Sound and Light.

THE MACMILLAN COMPANY,
NEW YORK: 66 FIFTH AVENUE.
CHICAGO: ROOM 23, AUDITORIUM.
SAN FRANCISCO: 327-331 SANSOME STREET.

VOLUME V NUMBER 4

The Astrophysical Journal

An International Review of Spectroscopy and Astronomical Physics

APRIL 1897

EDITORS

GEORGE E. HALE
Director of the Yerkes Observatory

JAMES E. KEELER
Director of the Allegheny Observatory

ASSISTANT EDITORS

J. S. AMES
Johns Hopkins University

HENRY CREW
Northwestern University

W. W. CAMPBELL
Lick Observatory

E. B. FROST
Dartmouth College

F. L. O. WADSWORTH, *Yerkes Observatory*

ASSOCIATE EDITORS

M. A. CORNU
École Polytechnique, Paris

C. S. HASTINGS
Yale University

N. C. DUNÉR
Astronomiska Observatorium, Upsala

A. A. MICHELSON
University of Chicago

WILLIAM HUGGINS
Tulse Hill Observatory, London

E. C. PICKERING
Harvard College Observatory

P. TACCHINI
Observatorio del Collegio Romano, Rome

H. A. ROWLAND
Johns Hopkins University

H. C. VOGEL
Astrophysikalisches Observatorium, Potsdam

C. A. YOUNG
Princeton University

CONTENTS

SPECTROSCOPIC NOTES . . . . . W. W. Campbell 233

ON THE SPECTRUM OF HYDROGEN . . . . . H. Kayser 243

THE CAUSE OF THE DARKNESS OF SUN-SPOTS . . . . . J. Evershed 244

THE YERKES OBSERVATORY OF THE UNIVERSITY OF CHICAGO. II. THE BUILDING AND MINOR INSTRUMENTS . . . . . George E. Hale 254

THERMAL MEASUREMENTS WITH THE BOLOMETER BY THE ZERO METHOD . . F. L. O. Wadsworth 268

MINOR CONTRIBUTIONS AND NOTES:

Note on Professor Campbell's Observations of Nova Aurigae; E. E. Barnard, 277.

Dr. Arendt's Spectroscopic Investigation of the Variation of Aqueous Vapor in the Atmosphere; Lewis E. Jewell, 279.

REVIEWS:

Physical Properties of X-Rays (C. W. Waidner), 282. *Ueber die ultra-violetten Funkenspectra der Elemente;* Franz Exner und E. Haschek, (H. C.), 290. *Color-Photography* (N. E. Dorsey), 291.

RECENT PUBLICATIONS 293–297

CHICAGO

The University of Chicago Press

WILLIAM WESLEY & SON, 28 Essex St., Strand, London
Sole Foreign Agents

SUBSCRIPTION, FOUR DOLLARS SINGLE NUMBERS, FIFTY CENTS

Die ergebenst unterzeichnete Verlagsbuchhandlung erlaubt sich, wiederholt auf die Bedeutung der

Beiblätter

zu den

Annalen der Physik und Chemie

unter Mitwirkung befreundeter Physiker

herausgegeben von

G. u. E. Wiedemann

hinzuweisen.

Die „Beiblätter", von denen bereits eine Reihe von 20 Bänden vorliegt, enthalten bekanntlich (jährlich 1600—2100) von Fachgelehrten bearbeitete Berichte über die gesamten im laufenden Jahr erschienenen Abhandlungen des In- und Auslandes aus dem Gebiete der Physik und ihren Grenzgebieten (physikalische Chemie, Astrophysik, u. s. f.). Ferner geben die Beiblätter monatliche Inhaltsangaben aller physikalischen Zeitschriften, Akademie-Publikationen und sonstiger schwer zugänglichen Arbeiten. In vielen Fällen haben die Referate den Verfassern vorgelegen und sind dadurch von ihnen autorisiert. Die Titel der in den „Annalen" veröffentlichten Arbeiten sind zur Herstellung einer vollständigen physikalischen Bibliographie in das Inhaltsverzeichnis aufgenommen.

Die Annalen und Beiblätter zusammen genügen demnach, um die physikalische Forschung aller Kulturvölker zu verfolgen; nur für specielle Fälle, wo Quellenstudien erforderlich sind, werden die Originalarbeiten stets unentbehrlich bleiben. — Ein vorheriges zu Ratheziehen der Beiblätter wird aber auch in diesen Fällen viel Mühe und Zeit ersparen.

Damit aber die erschienenen Arbeiten möglichst rasch besprochen werden können, erlaubt sich die ergebenst unterzeichnete Verlagsbuchhandlung im Auftrage der Redaktion an die Herren Gelehrten wiederholt die Bitte zu richten, von den in Zeitschriften erscheinenden eigenen Veröffentlichungen, sowie von denen ihrer Schüler der Redaktion zu Händen des Herrn Professor E. Wiedemann in Erlangen gefälligst sogleich nach Ausgabe einen Sonderabzug zuzusenden.

Abonnements *nimmt jede Buchhandlung, die Post und die Verlagsbuchhandlung entgegen. Der Bezugspreis für den Jahrgang von 12 Heften musste, des immer mehr wachsenden Umfangs wegen, von 1897 an auf 24 M festgesetzt werden. (Es enthält Jahrgang 1891 1424 Referate, dagegen Jahrgang 1896 bereits 2163.)*

— Die Bände 1—20 (1877—1896) sind in einzelnen completen Exemplaren noch zu haben. Ladenpreis 332 *M.* —

Hochachtend

die Verlagsbuchhandlung

Johann Ambrosius Barth.

Leipzig.

THE JOURNAL OF PHYSICAL CHEMISTRY

EDITED BY

WILDER D. BANCROFT JOSEPH E. TREVOR

PUBLISHED AT CORNELL UNIVERSITY

RECENT ARTICLES

On Irreversible Cells, I.	A. E. TAYLOR
Chemistry and its Laws	F. WALD
On Ternary Mixtures	W. D. BANCROFT
Allotropy of Sucrose	F. G. WIECHMANN
Nascent Hydrogen	R. FRANCHOT
On Irreversible Cells, II.	A. E. TAYLOR
Purification of Water	G. A. HULETT
Solution and Fusion	W. D. BANCROFT
Speed of Esterification	R. B. WARDER
Precipitation of Salts	H. A. BATHRICK
Inner Thermodynamic Equilibria	J. E. TREVOR
The Abegg Method, etc.	E. H. LOOMIS
Sulfur and Toluene	J. K. HAYWOOD
On Quintuple Points	W. D. BANCROFT
Solids and Vapors, Second Paper	W. D. BANCROFT
Osmotic Pressure and Variance	J. E. TREVOR

In addition to the original articles each number contains critical reviews of the current journal and book literature of Physical Chemistry.

ITHACA, N.Y.

THE EDITORS

LONDON
GAY AND BIRD
22, Bedford Street, Strand: W.C.
for the British Islands

LEIPZIG
BERNH. LIEBISCH
(K. F. Koehler's Antiquarium)
for the Continent of Europe

Issued every month except July, August, and September. Price $2.50 per year
The first volume will run to the close of 1897, and comprise twelve numbers

THE PHYSICAL REVIEW.

A Journal of Experimental and Theoretical Physics.

CONDUCTED BY

EDWARD L. NICHOLS, ERNEST MERRITT, and FREDERICK BEDELL.

XXI. November–December, 1896.

TABLE OF CONTENTS.

The Specific Heats of the Metals. F. A. WATERMAN.

Experimental Determinations of the Temperature in Geissler Tubes. R. W. WOOD.

The Viscosity of Mercury Vapor. A. A. NOYES and H. M. GOODWIN.

Minor Contributions: (1) An Example in Thermometry. *A. S. Cole and E. L. Durgan.* (2) A Study of the Apparent Capacity of Condensers for Short-charge Periods. *H. V. Carpenter.* (3) Note on the Osmotic Theory of the Voltaic Cell. *H. M. Goodwin.* (4) The Division of an Alternating Current in Parallel Circuits with Mutual Induction. *F. Bedell.* (5) On the Specific Gravity and Electrical Conductivity of the Normal Solutions of Sodium and Potassium Hydroxides, and Hydrochloric, Sulphuric, Nitric, and Oxalic Acids. *E. H. Loomis.*

Notes: (1) The Buffalo Meeting of the American Association for the Advancement of Science. (2) The British Association for the Advancement of Science.

New Books: *Wüllner:* Experimentalphysik. *Carhart:* Electrical Measurements. *Le Blanc:* Electrochemie. *Fleming:* Alternate Current Transformer. *Bedell:* Principles of the Transformer.

XXII. January–February, 1897.

TABLE OF CONTENTS.

The Freezing-points of Dilute Aqueous Solutions. III. E. H. LOOMIS.

A Method for Energy Measurements in the Infra-red Spectrum and the Properties of the Ordinary Ray in Quartz for Waves of Great Wave Length. ERNEST FOX NICHOLS.

Heat Rays of Great Wave Length. H. RUBENS and E. F. NICHOLS.

Minor Contributions: (1) On the Formation of Lead Sulphate in Alternating Current Electrolysis with Lead Electrodes. *Samuel Sheldon and Marcus B. Waterman.* (2) Polarization and Internal Resistance of a Galvanic Cell. *B. E. Moore and H. V. Carpenter.* (3) The Trace of the Gyroscopic Pendulum. *Ernest Merritt.* (4) Note on Different Forms of the Entropy Function. *W. F. Durand.*

New Books: *Frick:* Physikalische Technik. *Wilkinson:* Submarine Cable Laying. *Griesbach:* Physikalische Propaedeutik. *Weldon:* Physical Measurements.

Published Bimonthly. **Annual Subscription, Three Dollars.**

PUBLISHED FOR CORNELL UNIVERSITY.

THE MACMILLAN COMPANY - - - NEW YORK.

18-2-10